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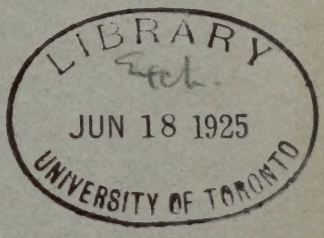
VOLUME II NUMBER 1

COLLECTED STUDIES AND REPORTS



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UNIVERSITY OF IOWA STUDIES IN PHYSICS

GEORGE WALTER STEWART, *Editor*

CONTINUATION OF CONTRIBUTIONS FROM THE PHYSICAL LABORATORY OF THE STATE UNIVERSITY OF IOWA

VOLUME II NUMBER 1

COLLECTED STUDIES AND REPORTS

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THE OPTICAL PROPERTIES OF AN ISOLATED CRYSTAL OF SELENIUM.

BY CHARLES H. SKINNER.

THE optical properties of various forms of selenium have been investigated by Meier,[1] Foersterling and Fréedericksz,[2] Gripenberg,[3] Pfund,[4] Wood[5] and others, but no results have yet been published on the optical properties of an isolated crystal of selenium. The present paper is to extend the investigations to this particular form.

The polarimetric, or Babinet compensator, method was used. The investigation was confined to a prism face of a crystal of the hexagonal prism type.[6] The reflecting surface was about 2 mm. by 5 mm., and required no polishing. A frequent cleansing in alcohol helped prevent the formation of surface films. Owing to the purity of such crystals, the surface may be taken as very probably representing the state of matter within the crystal.

The theory of metallic reflection for crystals of this type has been fully developed by Drude.[7]

Let $\overline{\phi}_1$, $\overline{\psi}_1$, be the angles of principal incidence and principal azimuth, respectively, when the crystal is in the first principal position (*i. e.*, when the principal axis of the crystal is parallel to the plane of incidence);

ϕ_2, $\overline{\psi}_2$, be the angles of principal incidence and principal azimuth, respectively, when the crystal is in the second principal position (*i. e.*, when the principal axis of the crystal is perpendicular to the plane of incidence);

n_1, n_2, k_1, k_2, R_1, R_2, be the indices of refraction, absorption coefficients, and coefficients of reflection, respectively, for the two principal positions:

[1] Meier, Ann. d. Phys., 31, p. 1029, 1910.

[2] Foersterling and Fréedericksz, Ann. d. Phys., 43, p. 1227, 1914.

[3] Gripenberg, Phys. Zeitschr., 14, p. 123, 1913.

[4] Pfund, Phys. Zeitschr., 10, p. 340, 1909.

[5] Wood, Phil. Mag., S. 6, Vol. 3, pp. 612–613, 1912.

[6] These crystals were obtained through the courtesy of Dr. F. C. Brown, of the physical laboratory of the State University of Iowa.

[7] P. Drude, Ann. d. Phys., 32, p. 613, 1887; 34, p. 515, 1888; 36, p. 542, 1889.

then,

$$\frac{\sqrt{\alpha}}{\cos \bar{\phi}_1} - \sqrt{\beta} \cos \bar{\phi}_1 = \cos 2\bar{\psi}_1 + i \sin 2\bar{\psi}_1;$$

$$\frac{\sqrt{\ }}{\cos \bar{\phi}_2} - \sqrt{\alpha} \cos \bar{\phi}_2 = \cos 2\bar{\psi}_2 + i \sin 2\bar{\psi}_2;$$

$$\alpha = a_{11} + ia_{12}; \qquad \beta = a_{21} + ia_{22};$$

$$\frac{a_{12}}{a_{11}} = \tan x; \qquad \frac{a_{22}}{a_{21}} = \tan \epsilon;$$

$$k_1 = \tan \frac{x}{2}; \qquad k_2 = \tan \frac{\epsilon}{2};$$

$$n_1^2 = \frac{2 \sin \frac{x}{2} \cos^3 \frac{x}{2}}{a_{12}}; \qquad n_2^2 = \frac{2 \sin \frac{\epsilon}{2} \cos^3 \frac{\epsilon}{2}}{a_{22}};$$

$$R_1 = \frac{n_1^2(1 + k_1^2) + 1 - 2n_1}{n_1^2(1 + k_1^2) + 1 + 2n_1}; \qquad R_2 = \frac{n_2^2(1 + k_2^2) + 1 - 2n_2}{n_2^2(1 + k_2^2) + 1 + 2n_2}.$$

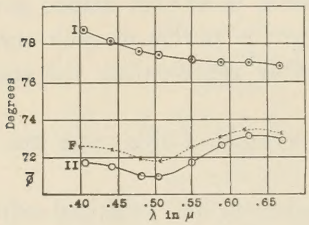

Fig. 1.

Principal Incidence.

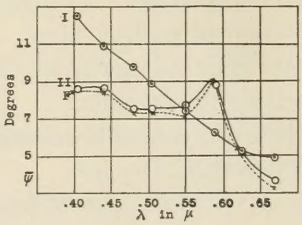

Fig. 2.

Principal Azimuth.

In the following table, the values of $\bar{\phi}$ and ψ are the means of several observations. The results are shown graphically in Figs. 1–5, where

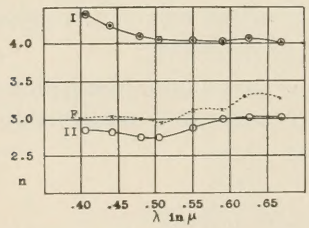

Fig. 3.

Index of Refraction.

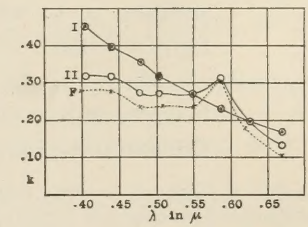

Fig. 4.

Absorption Coefficient.

$\bar{\phi}$, $\bar{\psi}$, and the calculated constants, are plotted with the wave-length. Curves marked I and II refer to the crystal in the first and second principal position, respectively.

It was found that when the crystal was placed in the second principal position, the results obtained agreed well with the best published results for crystalline selenium in mass. The curves marked *F* are from the results of Foersterling and Fréedericksz, who used mirrors obtained by spreading melted selenium over a well cleaned glass plate, and heating in a sand-bath at 180° C. Pfund determined *R* by a direct method, using mirrors obtained in the above manner. His results are shown in curve *P*, Fig. 5.

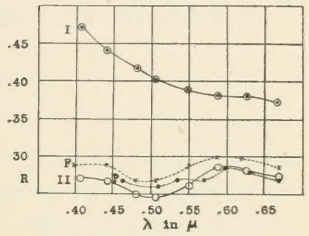

Fig. 5.

Reflecting Power.

With the crystal in the first principal position, quite different results were obtained, there being no similarity whatever to the results for the other position. It is interesting to note that the index of refraction as determined for this position is more than 15 per cent. greater than that found by any observer for other forms of selenium. There is no other known element with so high a refractive index.

The results show that isolated hexagonal crystals of selenium are doubly refracting. This points to the possibility that all such crystalline substances that reflect metallically have in general three sets of optical constants (two in any one plane); and that when one determines the

λ in μ.	First Principal Position.					Second Principal Position.				
	$\bar{\phi}_1$.	$\bar{\psi}_1$.	n_1.	k_1.	R_1.	$\bar{\phi}_2$.	$\bar{\psi}_2$.	n_2.	k_2.	R_2.
.406	78° 47′	12° 32′	4.40	.458	.472	71° 45′	8° 40′	2.86	.320	.273
.441	78 13	10 54	4.26	.395	.442	71 37	8 42	2.83	.319	.269
.480	77 34	9 51	4.12	.357	.419	71 07	7 35	2.77	.277	.252
.505	77 23	8 53	4.05	.317	.404	71 01	7 38	2.75	.276	.249
.550	77 11	7 31	4.05	.269	.393	71 46	7 42	2.87	.275	.264
.589	76 59	6 18	4.04	.229	.384	72 40	8 54	3.00	.315	.288
.626	77 01	5 20	4.09	.188	.383	73 13	5 16	3.17	.184	.285
.668	76 47	4 58	4.02	.172	.374	72 57	3 42	3.15	.132	.276

optical constants for any crystalline substance, used in mass, only a certain mean set of values is obtained.

In conclusion, I wish to acknowledge indebtedness to the staff of the physical laboratory of the State University of Iowa for the use of apparatus, and especially to Professor L. P. Sieg for suggesting the problem.

PHYSICAL LABORATORY,
STATE UNIVERSITY OF IOWA.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. IX, No. 6, June, 1917.]

THE SECONDARY INTENSITY MAXIMA IN BINAURAL BEATS.

BY G. W. STEWART.

THIS paper is a continuation of a consideration of binaural beats;[1] and contains the results of a study of one of the phenomena involved, namely, the secondary intensity maxima. The physics of hearing has been and is an important study, not because the discovery of new principles in physics are to be expected, but because the progress in the study of audition in the related sciences of psychology, anatomy, and physiology, is dependent upon a clear understanding of the physical aspects involved.

The reader should recall the fact[2] that when two tones of almost the same pitch are presented one to each ear, there appears an intensity maximum with 0° difference in phase, and secondary maxima at phase differences of $180° - \delta$ and $180° + \delta$. The secondary maxima are perceived clearly only when the beat-period exceeds one or two seconds. They seem to have an origin somewhat different than that of the zero phase maximum, and therefore a study of them is important.

The results herein presented were obtained with the apparatus already described.[3] There were two observers, one to operate the stop-watch at a chosen signal from the other who listened for the appearance of the secondary maxima. The method was to measure the time interval between the two secondary maxima, beginning with the first secondary maximum following zero-phase difference. Thus the interval included opposition in phase, and measured the time for a phase difference change of 2δ.

Results.—The secondary maxima will be considered as occurring at $180° - \delta$ and $180° + \delta$ phase differences, for to an observer watching the Lissajou's figures, it is evident that the phase difference of 180° occurs midway between the secondary maxima. The question arises, does δ change with the frequency of beats? The answer can be found in the following Table I.

[1] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, pp. 502–508.

[2] See *loc. cit.*

[3] *Loc. cit.*

TABLE I.

Forks of 128 d. v.

Date.	Period of Beats.	2δ in Sec.	2δ/2π.	2δ/2π.
Aug. 3	27.8		.20	
" 3	12.3	2.60(7)		.21
" 3	12.2	3.50(7)		.29
" 3	13.1	2.75(8)		.21
" 10	35.7	7.9 (4)	.22	
" 10	35.5	7.2 (6)	.20	
" 10	36.0	10.3 (9)	.29	
" 10	13.7	2.95(12)		.22
" 10	14.0	3.6 (11)		.26
" 11	35.2	7.9 (4)	.22	
" 11	13.3	2.68(10)		.20
" 11	13.5	3.1 (6)		.23
Mean.....	.....	.....	.23	.23

The number in parenthesis in the third column indicates the number of actual measurements, the average of which is expressed in the number just preceding. This table shows that with the beat-period increased by the factor three, no definite change takes place in δ. Similar results

TABLE II.

Forks of 42, 63, and 128 d. v.

Fork of 42 d.v.			Fork of 128 d.v.		
Date.	Beat Period.	δ/π	Date.	Beat Period.	δ/π
Aug. 11	20.4	.19 (10)	June 27	23.2	.25 (5)
" 11	20.2	.20 (8)	" 27	10.1	.29 (3)
Aug. 12	22.6	.18 (10)	June 29	23.9	.24 (3)
" 12	23.7	.16 (8)	" 29	31.0	.22 (3)
" 12	22.6	.18 (9)	" 29	16.0	.27 (4)
" 12	24.5	.15 (11)	July 2	35.8	.22 (4)
" 12	21.6	.17 (13)	" 3	27.8	.20
" 12	21.5	.18 (10)	" 3	12.3	.21 (7)
Mean.....	.....	.17	" 3	12.2	.29 (7)
Fork of 63 d.v.			" 3	13.1	.21 (8)
			" 10	36.0	.29 (9)
Aug. 3	10.1	.20 (11)	" 10	35.5	.20 (6)
"	10.7	.20 (13)	" 10	35.7	.22 (9)
"	10.8	.18 (11)	" 10	13.7	.22 (12)
Aug. 5	10.2	.18 (4)	" 10	13.7	.25 (10)
"	19.5	.16 (6)	" 10	14.0	.26 (11)
"	19.0	.14 (4)	" 10	13.6	.20 (7)
Weighted mean.....		.18	" 11	13.3	.20 (10)
			" 11	13.5	.23 (9)
			" 11	35.2	.22 (4)
			Mean	.....	.23

were obtained with forks of different frequencies, and can be inspected in succeeding tables where they are presented for another purpose. The data show the inaccuracy of the measurements of 2δ. A maximum would naturally be difficult to locate, for the hearing sense cannot detect small changes in intensity. But, as the succeeding data will show, this is not the only cause for inaccuracy. The constancy of δ with the frequency of beats is an important consideration in the theory of the secondary intensity maxima.

In what manner does δ vary with the frequency of the forks? Pairs of forks having approximately 42, 63, 128, 256, 384 and 469 d.v. were used in the experiments. The results are shown in Tables II. and III.

TABLE III.

Forks of 256, 361 and 469 d. v.

Fork of 256 d.v.			Fork of 361 d.v.		
Date.	Beat Period.	δ/π	Date.	Beat Period.	δ/π
July 13	13.2	.26 (9)	June 26	15.9	.28 (9)
" 13	12.2	.29 (9)	" 26	8.2	.29 (10)
" 14	43.8	.26 (4)	July 20	17.6	.28 (6)
" 14	27.4	.27 (8)	" 20	22.3	.26 (7)
" 14	23.4	.21 (8)	" 20	17.1	.25 (8)
" 14	12.0	.28 (11)	" 27	10.7	.27 (9)
" 14	10.5	.26 (9)	" 27	10.2	.24 (15)
" 14	9.25	.27 (9)	" 27	11.7	.30 (5)
Mean.....	.....	.26	" 27	9.1	.28 (9)
			" 27	12.5	.22 (7)
Fork of 469 d.v.			Aug. 1	12.7	.30 (6)
Aug. 5	4.2	.32 (1)	Mean.....	.....	.27
"	4.0	.37 (2)			
"	6.6	.27 (4)			
"	5.6	.35 (2)			
"	5.0	.38 (2)			
"	5.0	.26 (3)			
"	7.0	.27 (3)			
Mean.....	.....	.32			

Here in the third and sixth columns are given the values of δ as fractions of π. The number in parenthesis following each value gives the number of measurements of which that value is the average. The data indicate conclusively the inaccuracy of the measurement. They show that with the same forks and the same beat-frequency, but with observations made on different days, there is a considerable variation. It was found that observations made at different times of day with the observer in different physical states gave different results. Consequently, the observations

on each day were scattered as widely as possible. All of the observations recorded were made by the writer, assisted by Dr. Harold Stiles of the Iowa State College.

Fig. 1 is presented in order to show the character of the variation of δ with the frequency of the forks. While the results are not accurate, the fact that the relation is not linear is indubitable. δ increases less rapidly than in proportion to the frequency.

The reader should be reminded that in all of the observations herein reported the two tones were presented by means of stethoscope-binaurals which were inserted in the external meatus of the ear. Objections to this method are stated and answered in the previous article.[1] It is the opinion of the writer that while the quantitative results here presented might be different with open binaurals, yet the conclusions to be drawn from our experiments are not thereby affected.

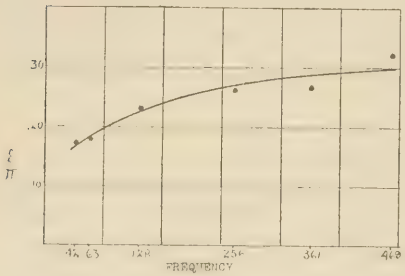

Fig. 1.

Conclusion.—A moment's consideration of the presence of the secondary maxima leads one to assume that the effect is due to conduction from one ear to the other through the head. The first result, viz., the fact that with a given fork frequency δ is independent of the beat-frequency, is distinctly in harmony with this assumption. For, suppose an indefinite number of conduction paths in the skull from the aërial vibration on one side to the ear of the other side. The resultant of these conducted waves will differ in phase from the original aërial vibration by an amount which depends upon the conducting paths and the frequency of the vibration. For any given fork frequency this introduction of phase is therefore constant. (It will be shown in the theory to be published in this journal that this phase is, in fact, δ.) Hence δ should be independent of beat-frqeuency. Thus the expectation that the explanation of the secondary maxima is to be found in physical theory receives encouragement.

The second result, which shows a nonlinear variation of δ with fork-frequency, might seem in disagreement with physical theory, for with any path between the ears, the retardation in phase by propagation thereon would be proportional to the frequency. This is true for a single path; it is true for a duplex path, but, as can be readily shown, only in

[1] G. W. Stewart, *loc. cit.*

case the amplitudes in these paths are equal. In as much as the amplitudes in all the conducting paths could not be equal, these paths cannot be equivalent to a single path of constant length. Hence we see that the two results herein described are not inconsistent with a theory which is based upon skull conduction from one side to the opposite organ of hearing.

I wish to acknowledge my indebtedness to Dr. Harold Stiles for his assistance in the observational work.

Physical Laboratory,
State University of Iowa.

BINAURAL BEATS.

[Reprinted from THE PHYSICAL REVIEW, N. S., Vol. IX, No. 6, June, 1917.]

BINAURAL BEATS.

BY G. W. STEWART.

THE first recorded experiments with binaural beats seem to have been made by Dove[1] in 1839. He found that beats were obtained with two forks in unison by separating the forks with one near each ear as well as by presenting both of them to one ear. Obviously these and indeed all binaural experiments have a considerable interest to the physicist, the physiologist, the anatomist, and the psychologist. This breadth of interest has resulted in a scattering of the literature and in serious omissions of recognition of earlier work. Paul Rostosky[2] has prepared an apparently thorough review of the experiments concerning the results obtained by and conditioned upon bilateral excitation of the organs of hearing. The review includes all results obtained up to and including the year 1896. The excellence of Rostosky's résumé enables the writer to abbreviate greatly references to the work therein described. The policy has been adopted in this paper of accepting results that are amply confirmed by relatively recent workers, without attempting to explain the earlier disagreements which were probably due to individual differences and to the limitations imposed by the particular forms of apparatus.

The chief phenomena observed when two tones differing in frequency are led one to each ear are as follows:[3]

(*a*) Beats occur whether the tones are presented to one ear or one to each ear. This was discovered by Dove,[4] observed subsequently by various experimenters and discovered independently by S. P. Thompson.[5] This phenomenon is observed by all, even by the partially deaf.

(*b*) With bilateral presentation of the tones, the minimum intensity observed in each beat does not (even with carefully adjusted equal intensities of the two sources) approximate zero. This is mentioned by S. P. Thompson[6] but doubtless was observed very much earlier by others.

[1] H. W. Dove, Repertorium d. Physik, Bd. 3, p. 404, 1839.

[2] Paul Rostosky, Beiträge zur Psychologie und Philosophie, I, 1905, pp. 173–273.

[3] Unless otherwise stated, the difference of frequencies is assumed to be small.

[4] *Loc. cit.*

[5] S. P. Thompson, Phil. Mag., 5, IV., 1877, p. 274.

[6] *Loc. cit.*

(*c*) When the tones at the ears are alike in intensity and phase, there is a localization in the median plane, the position in this plane varying with the observer and presumably with the apparatus arrangement. Purkyně[1] was the first to record this localization, which has been shown through numerous experiments of other observers, to be a usual accompaniment of binaural perception.

(*d*) With opposition in phase and equality in intensity, the sound is localized according to S. P. Thompson[2] at the back of the head. To several observers, with apparatus of a form different than that of Thompson's the localization is in the median plane. Doubtless the position in the median plane is influenced by the method of observation.

(*e*) The sound is localized on the side of the fork leading in phase when the phase difference is approximately 90°. Thus with phase differences the sound wanders from the median plane at 0° to one side at 90°, to the median plane at 180°, to the other side at 270° and again to the median plane at 360°. Perhaps S. P. Thompson[3] first observed the wandering from ear to ear but the periodic displacement of the localization was first most clearly described by Paul Rostosky.[4] Later workers, who were not aware of Rostosky's results, observed this lateral displacement. Lord Rayleigh[5] experimenting with forks, recognized the lateral effect and its relation to phase difference, and later[6] confirmed these observations with experiments conducted by the use of telephones. L. T. More and H. S. Fry[7] made preliminary experiments which were followed by those of L. T. More.[8] His results showed unmistakably the lateral displacement. More used an apparatus similar to that earlier adopted by C. S. Myers and H. A. Wilson.[9] A single source, branched pipes to the ears and an arrangement whereby the lengths of the pipes could be altered in opposite senses, were used. The experiments of Myers and Wilson also gave full confirmation of the lateral effects. Bowlker's[10] experiments do not seem to furnish reliable evidence. All the workers just mentioned were not aware of the splendid experiments of Rostosky published in 1902.[11]

(*f*) There is a limit to the frequency of the tone with which the

[1] Purkyně, Prager Vierteljahrsschrift, Bd. 67, p. 91, 1860.
[2] S. P. Thompson, Phil. Mag., 5, IV., 1877, p. 274, and Phil. Mag., 5, VI., 1878, p. 383.
[3] S. P. Thompson, Phil. Mag., 5, XII., 1881, p. 351.
[4] P. Rostosky, Philosophische Studien, 19, 1902, p. 557.
[5] Lord Rayleigh, Phil. Mag., XIII., 1907, p. 214.
[6] Lord Rayleigh, Phil. Mag., XIII., 1907, pp. 316–319.
[7] L. T. More and H. S. Fry, Phil. Mag., XIII., 1907, p. 452.
[8] L. T. More, Phil. Mag., XVIII., 1909, p. 308.
[9] C. S. Myers and H. A. Wilson, Proc. Roy. Soc., LXXX., 1908, p. 260.
[10] T. J. Bowlker, Phil. Mag., (6) XV., 1908, p. 318.
[11] *Loc. cit.*

effect just described (*e*) can be obtained. L. T. More[1] states that a fork of 512 d.v. is near to his limit of accuracy in the judgment by phase differences, that with 1,024 d.v. his judgment became untrustworthy and that with 3,000 d.v. he had no sense whatever of direction. Lord Rayleigh[2] gives the limiting frequency of his right and left sensations as 768 d.v. C. S. Myers and H. A. Wilson[3] state in general terms that with very high frequencies the lateral effects cannot be obtained.

(*g*) In addition to the maximum at 0° phase difference, there are also two additional maxima occurring one before and the other after opposition in phase, but distinctly perceptible only when the beats are less frequent than one in two to five seconds. These additional maxima were first studied by Paul Rostosky[4] in 1902, but in the succeeding literature no reference is made to his important contribution. Thus the subsequent discovery of these secondary maxima by the writer was an independent one. The results of the investigation of the latter are presented in this report.

(*h*) There are several other phenomena of importance, such as the effects obtained by placing forks on the skull, by using forks of large frequency differences, the result of practice of one ear upon the acuteness of the other, and the binaural audibility of two tones each one of which alone may be subliminal. For the present, we will be contented merely with this mention of them.

Apparatus.

Mirrors were attached on the prongs of tuning forks so that the plane of each mirror was perpendicular to the vibration of its prong. The forks were then mounted so that their vibrations were in perpendicular planes. The light from an arc lamp, by the use of lenses and an aperture, was reflected from two of the mirrors, one on each fork, and formed a small spot of light or an image of the aperture, upon a screen. With vibration of the forks, Lissajou's figures were traced upon the screen, and by these the phase relation of the forks could be ascertained. Glass tubes were mounted, one close to a prong of each fork and were connected by means of two pieces of rubber tubing each a meter in length, to ear tubes such as are used with stethoscopes. In certain experiments the manner of introducing the sound to the ear was changed, but of this mention will later be made. The forks were operated electrically by using a short helical spring contact, 0.3 cm. in radius, of platinum

[1] *Loc. cit.*

[2] Lord Rayleigh, Phil. Mag., XIII., 1907, p. 214.

[3] *Loc. cit.*

[4] *Loc. cit.*

wire, No. 29 B. & S. gage. The contacts were very quiet and gave a remarkable constancy of amplitude. Indeed the writer has observed the amplitude of a fork so actuated to remain constant, within one per cent., for an hour. No difficulty was experienced because of the presence of both forks in the same room. After many months' work with the apparatus, experimenting upon numerous individuals and confirming completely the experiments of Rostosky and others wherein even greater precautions were taken, the writer can confidently state that the results are in no wise dependent upon any "stray" sounds. Indeed, all the important results were confirmed by leading the tubes into an adjacent room where the forks were not at all audible to the unstopped ears. In practically all the experiments in the same room with the forks, the latter were scarcely, if at all, audible save when the tubes were placed in the ears.

The Results.

The following results were obtained with the assistance of Mr. F. C. Bruene.

1. The beats with the tones of two forks, one applied to each ear, were heard distinctly. Indeed, of twenty-three inexperienced observers listening for three periods of five minutes each, seventeen noticed the beats distinctly enough to report them. All experienced observers can hear them and record them with very satisfactory accuracy.

2. There existed a distinct wandering of the localization; in front at 0° difference of phase, on the side of the fork with the higher frequency leading in phase from 0° to 180°, and on the side of the slower fork with the higher frequency leading in phase from 180° to 360°, the changes of position being continuous, from front to rear (or within the head) and from rear to front. This wandering when first recognized might be described as occurring in a circular path. A more accurate description occurs in what follows.

3. This rotation of the localization just described was quickly and distinctly observed by some hearers; on the other hand all observers did not recognize it, even with an hour or more of practice. Eleven of the twenty-three uninstructed and inexperienced observers reported their observation of this "rotation" after fifteen minutes' trial (three trials of five minutes each). Six of these eleven noticed the effect within the first five minutes, and three in the second.

4. All of the eleven just mentioned, with the possible exception of one or two, noticed that the localization was much more distinct in the half cycle in which the lead of the faster fork varies from $-90°$ to $+90°$. With experienced observers, the effect is very noticeable.

5. Extended experiments with four experienced observers showed that the most accurate judgment as to phase difference, made by the localization, occurred when that difference is 0°.

6. With the experienced observers the change in the localization at 0° phase difference by the production of a very marked inequality in intensities, is slight. The rotation of the localization was modified by a slight circular shift in the direction of the side of the greater excitation. The effect upon the localization at the sides became very marked, that on the side of the greater intensity becoming more prominent and that on the other almost vanishing.

7. With observers of experience, but who had not yet recognized the secondary maxima,[1] the maxima intensity seemed to occur at 0° phase difference when the beat-period was less than perhaps two seconds. With a greater and gradually increasing period there arose an uncertainty until with a period of perhaps five seconds, the judgment of maximum intensity shifted to a phase difference noticeably less than 180°, or approximately to the first occurring secondary maximum.

8. The rotation of localization with experienced observers occurred as follows: At 0° phase difference the sound was localized as in front and distant, *i. e.*, external to the head. The localization then described in a horizontal plane a path which appeared to be somewhat circular, but in which the apparent distance contracted until the path entered the ear leading in phase. Then at about 180° phase difference the localization passed quickly through the head, into the other ear and from thence around to the front along a path symmetrical to the one just described.

9. When an experienced observer gave his attention to apparent intensity rather than to localization, there appeared three fairly distinct maxima. One occurred at 0° difference of phase, one at $180° - \delta$ and one at $180° + \delta$, δ being less than 45°. The earlier of the additional maxima at $180° - \delta$ phase difference, coincided with the localization in the ear near the higher frequency, and the later one with the localization in the other ear. With a 128 d.v. fork the secondary maxima seemed clearly discernible only if the beat-period exceeded a value of approximately two seconds. The value given is only approximate as the variation in the perception of the secondary maxima is continuous with the changing of the beat-frequency. The significant fact is that the 0° maximum was always present, whereas the secondary ones required a beat-period exceeding at least a second in order to become clearly evident.

10. The localization in the ears as described seemed to depend upon

[1] See (*g*) and (9).

the perception of the secondary maxima. If they were absent, as at high beat-frequency, the localization seemed confined to the region near the front. In other words the localization in approximately one half the cycle seemed to be due to the secondary maxima or the causes producing them.

Discussion of Results.

All of the results are not new. (1) was first discovered by Dove as already stated. (2) was in part reported by S. P. Thompson,[1] Lord Rayleigh,[2] C. S. Myers and H. A. Wilson,[3] and L. T. More.[4] Paul Rostosky was the first to record (2) and (8) and (9) completely. As previously stated in this article, Rostosky's work was unknown to Lord Rayleigh, C. S. Myers and H. A. Wilson, and L. T. More. Thus it chanced that the writer's discovery of the secondary maxima and the rotary character of the localization was an independent one.

None of the phenomena described owed its origin to sounds or noises other than the tones of the forks. Neither did any sound enter the left ear by aerial conduction from the tube entering the right ear. The former statement was tested by conducting the tubes to an adjoining room and by the many alterations in apparatus necessitated by the changing of the forks in experiments described in a succeeding article. The latter statement was verified by the use of tones of very faint audibility and by changing the character of the introduction of sound to the ear (*i. e.*, by the use of binaurals that are not inserted in the ears), without changing the phenomena observed in either case. Reference to Rostosky's experiments will show that he took much more elaborate precautions. Yet our results are in agreement with his at every point. The evidence thus favors the judgment that our precautions were sufficient.

There is an objection to the use of binaurals inserted in the ears, for by partially closing the external meatus they emphasize sounds that are conveyed from the skull to the volume of enclosed air. From the physical point of view, this partial closing of the external meatus would not be likely to change the characteristics of the phenomena and experiment shows this to be the case. Indeed, Rostosky used binaurals that were not inserted into but merely presented in front of the opening of the ear. The writer was also able to detect all the phenomena without the use of inserted binaurals. But there is an additional objection to the inserted binaurals, for there is the chance of conduction from the terminal of the binaural to skull. Experiments in stopping the openings

[1] *Loc. cit.*
[2] *Loc. cit.*
[3] *Loc. cit.*
[4] *Loc. cit.*

of the binaurals showed that there remained only an exceedingly faint trace of sound which could be fully explained by the failure of the stoppage. The conclusion of the writer is that the results with inserted and with non-inserted binaurals are not qualitatively different.

The results given point definitely to distinct differences between the causes of the localization in front and in the rear, or in the ears. The two halves of the cycle differ in distinctness of localization, in independence of beat frequency, in effects produced by inequalities of intensities, and in the apparent distance from the head, the localization in one half being external and in the other half internal. These differences must be accounted for in any adequate theory.

The fact that some observers by the single source method of C. S. Myers and H. A. Wilson[1] do not obtain such a clearly defined "rotation" but rather a wandering back and forth from the median plane to the sides, is apparently due to the difference in the method of observing. In our experiments, with the continually changing phase difference, the rotation is observed in every case where localization is obtained.

This article gives the chief phenomena characteristic of binaural beats. In succeeding articles will be presented a study of the secondary maxima and a physical theory of all the phenomena involved. It will appear that the experiments do furnish excellent evidence as to the perception of phase by the ear and as to a two-fold nature of the hearing organ.

I wish to acknowledge my indebtedness to Mr. F. C. Bruene for his assistance in the experimental work.

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[1] *Loc. cit.*

THE THEORY OF BINAURAL BEATS

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THE THEORY OF BINAURAL BEATS.

BY G. W. STEWART.

AS described in preceding articles on binaural beats,[1] there are several phenomena that invite an explanation in terms of physics. The three outstanding aspects are (1) the existence of the "binaural" beats, herein referred to as "primary" beats which correspond to these heard uniaurally with two forks almost in unison, (2) the distinct perception of secondary intensity-maxima when the binaural beat-period is greater than two to five seconds, and (3) the wandering localization of the sound during each beat. We will consider these three separately and propose a theory which seems to promise a satisfactory explanation.

PRIMARY BEATS.

Dove[2] in 1839 discovered that when two forks, almost in unison, were presented first to one ear and then, by carrying one about the head, one to each ear, the beats perceived in the first position disappeared and subsequently reappeared as the moving fork approached its final position. I have chosen to call these binaural beats the "primary" beats, for they are always present with observers who are not seriously deaf in one ear. This is not true of the "secondary" beats which occur with clearness only if the primary beat-period exceeds two to five seconds. Separate treatment of the primary beats is justifiable, and in order to make the theory conform strictly to the fact, let us suppose we are dealing with a primary beat-period of approximately one second, or a period where the primary beats alone appear. As already described,[3] we have then a maximum with 0° and a minimum with 180° phase differences.

Let us assume that the forks F_1 and F_2 are presented one to each ear, that their amplitudes are equal, that F_1 has the greater frequency, and that there are conducting paths from each fork to the farther ear.

Let ϵ_1 be the retardation in phase in transmission from each fork to the nearer ear. (By the "ear" is meant the physical instrument producing the sensation.)

[1] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, p. 502 and p. 509.

[2] H. W. Dove, Repertorium d. Physik, Bd. 3, p. 404, 1839.

[3] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, p. 502.

Let ϵ_2 be the phase retardation in transmission to the farther ear.

Let θ be the phase of the slower fork. $\theta = 2\pi ft$, where f is the frequency.

Let $\theta + \epsilon$ be the phase of the faster fork.

Let α be the amplitude at the ear produced by the nearer fork.

Let β be the amplitude at the ear produced by the farther fork.

The the resulting displacement at the ear nearer to the faster fork, at the instant represented by the value θ, is $\alpha \cos(\theta+\epsilon-\epsilon_1)+\beta \cos(\theta-\epsilon_2)$. At the farther ear the displacement is $\alpha \cos(\theta-\epsilon_1)+\beta \cos(\theta+\epsilon-\epsilon_2)$. By expanding we find the former displacement to be $\sin\theta[-\alpha \sin(\epsilon-\epsilon_1)+\beta \sin\epsilon_2]+\cos\theta\,[\alpha \cos(\epsilon-\epsilon_1)+\beta\cos\epsilon_2]$ and the maximum displacement, or amplitude, to be

$$\sqrt{[-\alpha \sin(\epsilon-\epsilon_1)+\beta \sin\epsilon_2]^2+[\alpha\cos(\epsilon-\epsilon_1)+\beta\cos\epsilon_2]^2}.$$

The energy of the vibration is proportional to the square of the amplitude. Hence the energy at the ear nearer to the faster fork is,

$$E_1 = \alpha^2+\beta^2+2\alpha\beta\cos(\epsilon-\epsilon_1+\epsilon_2).$$

E_2, or the energy at the ear farther from the faster fork, can be written down by interchanging β and α and ϵ_2 and ϵ_1, or,

$$E_2 = \alpha^2+\beta^2+2\alpha\beta\cos(\epsilon-\epsilon_2+\epsilon_1).$$

Let $\epsilon_2-\epsilon_1$ be represented by λ.

Then

$$E_1 = \alpha^2+\beta^2+2\alpha\beta\cos(\epsilon+\lambda) \quad (1)$$

$$E_2 = \alpha^2+\beta^2+2\alpha\beta\cos(\epsilon-\lambda) \quad (2)$$

If the apparent intensity is determined by the sum of E_1 and E_2, as we will assume, then,

$$E_1+E_2 = 2(\alpha^2+\beta^2)+4\alpha\beta\cos\epsilon\cos\lambda \quad (3)$$

and the apparent intensity will be a maximum at $\epsilon = 0°$ and a minimum at $\epsilon = 180°$. Moreover, since in the very nature of the case β is less than α, the minimum can never be a silence. Thus the theory coincides qualitatively with experiment. Quantitative comparison seems impossible.

Discussion.—It will be observed that the above theory does not designate the points of interference in each organ of hearing, but that it does stipulate the occurrence of the interference in a wholly physical instrument. In other words, peripheral interference is accepted as the cause of the primary maxima, or the "beats." To this extent the theory is not new, for Dove,[1] the discoverer of the primary maxima,

[1] *Loc. cit.*

recognized peripheral interference as a possible explanation. This suggestion has often been repeated. Paul Rostosky,[1] in his theory of the localization and the secondary maxima accepts this non-synchronous variation of the resulting intensity in each ear. But neither in his excellent review[2] nor in his own contribution just mentioned does Rostosky give a detailed theory explaining the primary maxima and minima.

There may be an objection to the summation of intensities herein employed, but in justification the writer submits four considerations. First, we have the fact that two like tones, each alone faint enough to be just at the limit of hearing, become distinctly audible when simultaneously each is presented to an ear.[3] Second, summation seems reasonable when one considers the phenomena of the strengthening of intensity in the numerous binaural experiments recorded in Rostosky's review, just mentioned. Third, the experiments of the writer[4] upon the accuracy of localization of various frequencies, the head not remaining stationary, gives strong evidence in favor of the intensity-sum as an important factor. In these experiments the frequencies most accurately localized are those in which the theoretical variations of the intensity-sum are the greatest. Finally, the perfect blending of the two tones which is actually observed points distinctly to the summation. According to the theory just proposed, the effect is independent of beat-frequency, that is, the primary maxima and minima ought always to appear, which is in accord with fact. The above simple theory would seem to explain the primary maxima and minima satisfactorily. But the phenomena is much more complex, and these additional complexities will be discussed in a following section.

Peterson[5] discusses the "nature and probable origin" of primary beats and concludes that they are cortical rather than peripheral in origin. His reasons are based upon the evidence of Cross & Goodwin[6] to the effect that difference tones cannot be obtained binaurally, the evidence, from experiments on two people each deaf in one ear, that bone conduction must be very small, and upon certain experiments in the counting of the primary beats. The evidence offered by Peterson and the argument in favor of his conclusion are worthy of careful consideration. But in as much as he does not set up any physical theory, a discussion of his paper is probably not appropriate to this article. But the writer should

[1] P. Rostosky, Philosophie Studien, 1902, 19, p. 557.
[2] P. Rostosky, Beiträge Zur Psychologie and Philosophie, I., 1905, pp. 173–273.
[3] Tarchanow, St. Peterburger Med. Wochensclos., 1878, No. 43, (n.R.).
[4] G. W. Stewart, PHYS. REV., N.S., Vol. 2, No. 1, July, 1913.
[5] Joseph Peterson, Psychological Review, Vol. XXIII, No. 5, Sept., 1916.
[6] C. R. Cross & H. M. Goodwin, Proc. of Academy of Arts & Science, 1891, 27, 1.

call attention to the fact that the theory of this paper not only assumes a peripheral interference which Peterson denies, but in addition a summation of intensities or a blending which obviously cannot be peripheral.

The Secondary Maxima.

The only records of the appearance of the secondary maxima are those of Paul Rostosky[1] and the writer.[2] Rostosky seeks to explain these maxima which appear first in one ear and then in the other, by a consideration of the localization. He concludes that the ratio of the two intensities, represented by equations (1) and (2), determines the degree of the right or left localizations. If one assumes α and β to be of the same order, say 1 to $\sqrt{2}$ or 1 to 2 (the values used by Rostosky), and then plots the ratio of E_2, (2), to E_1, (1), he will obtain a curve similar to Fig. 1. The curve is interpreted by Rostosky as the degree of right localization, the lower half of the curve, inverted, being the degree of left localization. Thus the maximum right localization appears distinctly or extremely right and gives the impression of a maximum intensity. Hence the two secondary maxima occurring in the neighborhood of a difference of phase of 180° are explained qualitatively; indeed, there also seem to be no objections thereto on the basis of quantitative considerations. Unfortunately, however, Rostosky makes a fatal error in his theory. His equations similar to (1) and (2) are[3] as follows:

$$J_l = a^2 + a_1^2 + 2aa_1 \cos (x + d/2)$$
$$J_r = a^2 + a_1^2 + 2aa_1 \cos (x - d/2).$$

Where the subscripts l and r refer to the left and right sides respectively, and a, a_1, x and $d/2$ correspond to our α, β, ϵ and λ, respectively. d is a positive quantity, hence J_l reaches a maximum, with changing phase x before J_r, the former occurring at $x = -d/2$ and the latter at $x = d/2$. The faster fork is thus on the left. Moreover, between $x = 0°$ and $x = 180°$, J_l is less than J_r. Experimentally, the secondary maximum appears first (after $x = 0°$) on the side of the faster fork, and hence on the side of the less theoretical intensity. If the maximum in Rostosky's curve, Fig. 1, represents localization, then (accepting his equations just given) it must represent "left" localization. But this curve is the ratio J_r/J_l and therefore must be interpreted as "right" localization. Hence we find that experiment and his theory do not agree. His error is due to a confusion of sides; in fact, one is disposed to guess (incorrectly) that the side of greater intensity (when $0° < x < 180°$) is that of the faster fork.

[1] Paul Rostosky, Philosophie Studien, 1902, 19, p. 557.

[2] G. W. Stewart, Phys. Rev., 2d Series, Vol. IX., No. 6, 1917, 509–513.

[3] See *loc. cit.*, p. 580.

The writer has considered whether or not an additional assumption of a change in phase during the conduction across the head would lead to a revision of Rostosky's theory that would be in harmony with observation, and he finds a revision upon such an assumption impossible. We are thus led to a complete rejection of the theory of Rostosky.

The writer's theory of the secondary maxima is based upon the outstanding importance of the following experimental facts which have been described in the two preceding articles.[1]

1. The secondary maxima do not occur unless the period of the beats is greater than, say, two to five seconds, whereas the presence of the primary maxima is independent of the beat-period.

2. At the secondary maxima the sound seems localized in the ears (first in the one nearer the faster fork), and between the maxima within the head. With the primary maxima the localization is distinctly external to the head.

3. The sound seems to be much rougher at the secondary than at the primary maxima.

4. The movement of the localization is much more rapid between the two secondary maxima than elsewhere in the rotation.

5. If the secondary maxima appear at phase differences of $180° - \delta$ and $180° + \delta$ then, for a given fork pitch, δ is independent of the beat-frequency, but δ does vary with the frequency of the forks used, and this variation is not a linear one.

6. Unequal exciting intensities at the ears have a tendency to eliminate the secondary maximum on the side of the weaker source.

7. When the beat-frequency is much more rapid than one per second, not only do the secondary maxima disappear but localization in the region of 180° phase difference also vanishes.

These facts give a definite indication of the difference in origin of the primary and secondary maxima. But before we admit a complexity to account for this difference in origin, let us consider a physically simple equivalent of the skull and ear organs. Let us assume two physical instruments capable of recording vibrations and connected by numerous sound conducting paths. Is it possible for such an arrangement with two beating tones presented simultaneously one to each ear, to make a record that will show the three maxima and three minima actually experienced in each beat-cycle? Let us first assume that each instrument records intensity or a value proportional to the square of the amplitude at the instrument and that the resulting combined record is proportional to the arithmetical sum of these intensities. Upon this assumption we

[1] G. W. Stewart, loc. cit.

have already found, equation (3), that we may expect only a maximum at $\epsilon = 0°$ and a minimum at $\epsilon = 180°$.

If one assumes that the ear is an instrument that records not intensity but the time integral of the absolute value of the displacement, or $\int_0^{2\pi} |\text{displacement}|\, d\theta$,[1] he finds that the record in the nearer ear is proportional to

$$|\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \cos(\epsilon - \epsilon_1 + \epsilon_2)}|$$

and in the farther ear to

$$|\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \cos(\epsilon + \epsilon_1 - \epsilon_2)}|.$$

Reference to the equations just preceding (1) will show that these values are, in fact, the absolute values of the amplitudes. The total record, or the blended sensation, would be proportional to the sum of these two absolute values. It can be shown that if $\cos(\epsilon_2 - \epsilon_1)$ is greater than $2\alpha\beta/(\alpha^2 + \beta^2)$ this sum will be a maximum at 0° and a minimum at 180°. Obviously, $\cos(\epsilon_2 - \epsilon_1)$ is nearly unity, whereas the term $2\alpha\beta/(\alpha^2 + \beta^2)$ is, in all likelihood, much less than unity. We conclude that the foregoing assumption as to the dependence of the sensation upon the absolute value of the amplitude at the physical instrument leads to the same maximum and minimum as that based upon the dependence of the sensation upon the intensity of vibration. This leads one to doubt whether any assumption can be made as to the record of the instrument, the ear, which will result in a total sensation that has three maxima and three minima in one beat-cycle. At any rate, lacking such an assumption, the conclusion is that we must consider the ear not a single instrument, even for one tone. If, for our present purpose, one can consider each ear to consist of two different physical instruments which act somewhat independently of one another, a distinct gain is made thereby. For, assuming the sensations to be non-interfering, it is obvious that the sensations possible with two somewhat independent instruments are very different than those possible with a single instrument. For, suppose that instrument number one vibrates with the resulting vibration $f(t)$ giving the appropriate sensation, and that instrument number two vibrates with the resulting vibration $F(t)$, giving its appropriate sensation. If the sensations are non-interfering, the total sensation cannot, in general, be the same as had the vibrations $f(t)$ and $F(t)$ been combined in a single instrument.

Our theory will assume the equivalent of two physical instruments

[1] The writer is indebted for the suggestion of this assumption to Professor R. P. Baker, of the Department of Mathematics in the State University of Iowa.

in each ear excited in a mechanically different manner. A pair of like instruments, one on each side, is responsible for the primary maxima and minima, and another pair for the secondary maxima and minima. The former pair is excited chiefly through the ear drum-skin. The latter is excited chiefly through the vibrations of the skull. We will assume the mechanical arrangement to be such that the vibration conveyed by the skull from one fork to the distant ear has its phase determined not only by the equivalent length of path, but also by a change in phase of 180°. We will first discuss the consequence of the foregoing assumptions, and then its agreement with experiments.

The pair of A instruments, assumed excited chiefly through the membrana tympani, will give the primary maxima and minima provided we can also assume a conduction from one ear to the other. This conduction can occur by many paths and has been demonstrated actually to exist,[1] but the methods used could not exhibit the excellence of the conduction. We will assume this conduction as a fact without attempting to describe it in detail. Thus our previous theory, equation (3), can be regarded as applicable to the A instruments, and the primary maxima are completely explained.

The intensities in the B instruments can best be understood by reverting to the equations from which (1) and (2) were obtained. We will use α_1, β_1, E_1', E_2', ϵ_2' and ϵ_1' instead of α, β, E_1, E_2, ϵ_2 and ϵ_1 respectively, inasmuch as the values for the B instruments are different. Remembering the assumption that ϵ_2 really becomes $\epsilon_2' + 180°$ we obtain the displacement at the nearer ear, $\alpha_1 \cos(\theta + \epsilon - \epsilon_1') + \beta_1 \cos(\theta - \epsilon_2' - 180°)$ and at the farther ear, $\alpha_1 \cos(\theta - \epsilon_1') + \beta_1 \cos(\theta + \epsilon - \epsilon_2' - 180°)$. The intensities at the nearer and farther ears become, substituting λ' for $\epsilon_2' - \epsilon_1'$,

$$E_1' = \alpha_1^2 + \beta_1^2 - 2\alpha_1\beta_1 \cos(\epsilon + \lambda') \quad (4)$$

$$E_2' = \alpha_1^2 + \beta_1^2 - 2\alpha_1\beta_1 \cos(\epsilon - \lambda'). \quad (5)$$

Taking the sum of intensities as before, we have,

$$E_1^2 + E_2^2 = 2(\alpha_1^2 + \beta_1^2) - 4\alpha_1\beta_1 \cos\epsilon \cos\lambda' \quad (6)$$

This sum has a maximum at 180° and a minimum at 0°. If the total sensation varies as the sum of equations (3) and (6), or the total intensity sum of both pairs of instruments A and B, then, no matter what values are assigned to our constants, we could obtain only one maximum and one minimum for one beat-cycle. This can be shown by the usual method of determining maxima and minima. But the assumption that

[1] K. L. Schaefer, Pflüg. Arch., Bd. 61, p. 544, 1895, and also C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 1908, A 80, p. 260.

insturments A and B are differently excited, leads us to assume also a difference in sensation from these excitations. If α_1 and β_1 have nearly the same value the intensity sum of the B instruments will be a maximum at 180° and a minimum, almost zero, at 0° phase differences. We would expect, then, the resulting observed effects at 0° to be chiefly the intensity-sum produced by the A instrument and at 180° to be the combination of the minimum of the A-intensity-sum and the somewhat different maximum of the B-intensity-sum. This is precisely the impression that an observer first obtains. He does not find two secondary maxima but only one in the neighborhood of 180° phase difference. Further, this maximum at 180° as already stated, appears to come from a source located in or on the skull, whereas the maximum at 0° is externally localized. This effect is in entire agreement with the assumption that the B instruments are excited by a skull vibration and the A instruments by an aërial vibration in the external meatus. The fact that the 180° maximum does not appear unless the beat-period is longer than two to five seconds indicates that the excitation of the B instruments requires a longer time, or that the inertia of the system is large, and this is in harmony with the assumption just mentioned.

But how account for two secondary maxima? In discussing the addition of intensities with the A instruments, we tacitly assumed a blending of the sound so that the maximum in either instrument was not noticed. This blending is, in fact, a part of our usual experience. But we have no reason for believing that the usual degree of blending must occur with the intensities at the B instruments. Indeed, the cause of blending (with the A instruments) is not understood. We may, therefore, without making any additional assumptions investigate what the effect would be if the observer is able to recognize the maximum intensity in each B instrument. Equations (4) and (5) show that the maximum intensity in the B instrument located on the side nearer the faster fork occurs before the maximum in the other B instrument by a phase of $2\lambda'$. Therefore, the maximum at the nearer B instrument would occur at $180° - \lambda'$ and at the farther B instrument at $180° + \lambda'$ phase differences. Further, the localization, because the excitation is from the skull, would appear to be first in the nearer and then in the farther ear. Again the theory is in distinct harmony with experiment. Thus, by assuming a difference between the A and B instruments in the manner of excitation and a difference in the degree of blending of the sensations, we can account for the appearance of the secondary maxima.

The difference in localization in the two parts of the beat-cycle, near 0° and near 180° phase difference, will be discussed in detail on a succeed-

ing page, but the comment may here be made that the cause is to be found in the difference in the manner of excitation of the two pairs of instruments.

The difference in the roughness of the sound in the two parts of the beat-cycle may also be explained by differences in excitation, for the excitation via the membrana tympani does, as we know, enable us to detect purity of tone, whereas the excitation directly by the skull would undoubtedly include natural vibrations of the skull itself and thus produce a somewhat rougher sound.

The rapid movement of the localization will also be presently discussed and satisfactorily explained on the basis of our theory.

The fact that δ (or λ' in our theory) is independent of the beat-frequency meets ready explanation for λ' represents the difference in retardation due to difference in the paths to the nearer and farther B ear, and certainly this value depends only upon the frequency of vibration and the conducting paths and not upon the rate at which the phase differences of the sources are changing. Or, in other words, λ' is entirely independent of ϵ and $d\epsilon/dt$.

But how does theory agree with the variation in $2\lambda'$ (our earlier 2δ) with frequency, as shown in Fig. 1 of a previous article?[1] We have assumed a retardation in phase of ϵ_1' in the propagation from the source

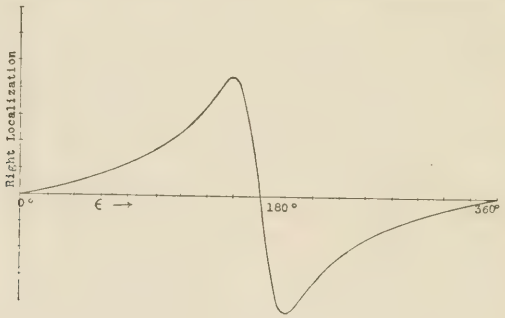

Fig. 1.

to the nearer instrument, and a retardation of ϵ_2' in the path from the source to the farther instrument. The excess retardation in conduction to the farther ear is thus $\epsilon_2' - \epsilon_1'$ or λ'. We can now introduce a helpful simplification without sensible error. Assume ϵ_1' occurs in a single path. Then it will be proportional to the frequency. But ϵ_2' will be a resultant of conduction over an indefinite number of paths, with varying amplitudes. Can ϵ_2' be regarded as occurring in an equivalent single path

[1] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, pp. 507–513.

and hence be considered as proportional to frequency? As we will see, the answer is a negative one. For let us consider two paths only, along which the vibrations have the amplitudes a_1 and a_2. The difference in time required to transverse these paths is constant and hence the difference in phase at the distant junction of the paths is proportional to the frequency. Let this phase difference be γ, let r be the resultant amplitude, and let ϵ' be the phase difference between the resultant vibration and the one proceeding along the longer path, say, a_2.

Then

$$r^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \gamma \tag{7}$$

and

$$r \sin \epsilon' = a_1 \sin \gamma \tag{8}$$

From these equations we get, in case $a_1 = a_2$,

$$2 \cos \tfrac{1}{2}\gamma \sin \epsilon' = \sin \gamma$$

or

$$\epsilon' = \gamma/2.$$

Thus with equal amplitudes, or with $a_2 = 0$, ϵ' is proportional to γ and therefore to the frequency. The assumption of equality of amplitudes would therefore give us a change in the phase, ϵ_2', that is proportional to the frequency. But the assumption of equality of amplitude is not correct, for different paths would, on account of the differences in length and the presence of damping, have different amplitudes. We are then compelled to reject the suggestion that the various paths can be equivalent to a single path. Our expectation would therefore be that λ' or $\epsilon_2' - \epsilon_1'$ would change either more rapidly or less rapidly than in proportion to the frequency. Our experiments, as already described, show the latter to be the case.

As already stated, if the intensities of the two sources are far from equal, the general effect of a maximum in the neighborhood of 180° phase difference is retained, but the secondary maximum on the side of the less intensity is very noticeably lessened. This is apparently caused by the fact that there is enough blending of tone to cover up the individual maximum which is, as experiment shows, only recognized at all with practice.

We have thus discussed all the phenomena (excepting those presently to be considered under "localization") and have found that all are distinctly in harmony with the theory herein presented.

The Localization.

Lord Rayleigh[1] is inclined to favor perception of phase difference as one of the explanations of the phenomena of localization. At zero-phase

[1] Lord Rayleigh, Proc. Roy. Soc., 1909, A 83, pp. 61–64.

difference the sound is localized in front from which point it seems to wander about the head in the direction of the side of the tone of higher frequency, then into the ear. It then seems to pass suddenly (at 180° phase difference) through the head, reappearing in the other ear, and then back to the median plane in front by a path similar to that of the first half of the beat-cycle. C. S. Myers and H. A. Wilson[1] from experimental evidence showing the variation of the displacement of the localization with changing phase, accept the view that the lateral displacement is proportional to the difference of the intensities at the ears. If we subtract (2) from (1) we get $-4\alpha\beta \sin\epsilon \sin\lambda$. If now we assume a phase change of 180° to be introduced in transmission we get, by substituting $\lambda + 180°$ for λ, $+4\alpha\beta \sin\epsilon \sin\lambda$. The lateral displacement is then proportional to $\sin\epsilon$ and is zero when $\epsilon = 0°$ or 180° and is a maximum for $\epsilon = 90°$. This is the Myers-Wilson theory of localization. It does offer an explanation of the wandering of the sound toward and away from the median plane, but it is objectionable because by using this physical theory one cannot explain the primary maxima, the secondary maxima, the entrance of latter only when the beat-period is sufficiently long, the external localization in the region, $\epsilon = 0°$, the internal localization in the region $\epsilon = 180°$, the very small effect of excessive inequalities in intensities upon the localization in the region $\epsilon = 0°$ and the marked effect in the region $\epsilon = 180°$, the difference in rapidity of movement in the localization at $\epsilon = 0°$ and $\epsilon = 180°$, and the simultaneous disappearance of localization and secondary beats in the region $\epsilon = 180°$, when the primary beats are much more rapid than one per second.[2] In fact, the localization in the half cycle $\epsilon = 270°$ to $2\pi + 90°$ differs so much from the localization in the other half that one can hardly accept a theory that does not also make a distinction. In justice to Messrs. Myers and Wilson the statement should be made that their form of apparatus did not lead them to perceive most of the features that have been mentioned.

In the view of the writer the explanation of the localization is not so important as that of the primary and secondary maxima, for while the localization does not occur with all observers even with considerable practice, the presence of the maxima seems common to all. From the physicist's viewpoint, our theory would be that in the phase region, $270° < \epsilon < 2\pi + 90°$, difference of phase at the A instruments[3] is a

[1] C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 1908, A 80, p. 260.

[2] Results given by G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, pp. 502–508.

[3] For our present purpose it maybe sufficient to note that the phase of the fork nearer to an ear will furnish the greater portion of the amplitude and thus that the variation in phase difference at the A instruments is largely determined by the phase difference of the forks themselves.

determining factor, and that in the region $90° < \epsilon < 270°$ the non-blending excitation of the B instruments, first with greater intensity on the side of the higher pitch, and then with equal intensity and then with greater intensity on the other side, explains the localization. The explanation of the region $270° < \epsilon < 2\pi + 90°$ by phase difference will not meet the approval of many psychologists, on the ground that sensation depends upon the mode of response of the end organ and not upon the character of the stimulus. But one must not treat too seriously the application of a generalization that has never been definitely established. Moreover, inasmuch as all observers do not obtain the variation of localization herein described, we may assume it to be a "second-order effect" or one whose cause may never have been detected in experiments with sensory stimuli. Peterson[1] considers the localization in this region as caused by phase differences and concludes that the perception of phase difference is cortical in origin. The explanation given above of the localization in the region $90° < \epsilon < 270°$ does not meet with the objection just cited or any other of which the author is aware. Moreover, it should be anticipated that the excitation of the A instruments by the usual route, membrana tympani, etc., would give an external localization and the excitation of the B instruments, directly by the skull, an internal localization, just as is always produced by any vibration given the skull only. The effects produced by unequal intensities at the ears also accord with the theory. For if the localization near $\epsilon = 0°$ is due chiefly to a perception of phase difference by the A instruments the effect of unequal intensity should be slight. Again, if the localization near $\epsilon = 180°$ is caused by the lack of complete blending of the intensities at the B instruments, it would be seriously modified by inequality in the intensities at the ears. Both of these conclusions are verified by experiment. The rapid movement of the localization in the region of 180° is in accord with the theory, which gives the variation in phase difference with localization in the ears as $2\lambda'$ or $2(\epsilon_2' - \epsilon_1')$, for, from physical considerations, we can see that $\epsilon_2' - \epsilon_1'$ must be a small angle. Suppose the equivalent difference of path in the skull bones were 20 cm. If this were in air, the phase introduced thereby for a frequency of 256 d.v. would be about 60°. But the value λ' or δ for this frequency is approximately 45°.[2] Inasmuch as the velocity in the skull bones must be several times[3] that in the air, we find that the theory would suggest an angle even smaller than that found by experiment. Both theory and experiment agree qualitatively that the dif-

[1] J. Peterson, *loc. cit.*

[2] G. W. Stewart, PHYS. REV.

[3] H. Frey, Ztschr. f. Psych. u. Phys. d. Sinnesorgane, Bd. 28, p. 9, 1902, gives *ten times*.

ference between the phase relations at the two secondary maxima is markedly less than 180° and hence that the passage from the first secondary maximum (following 0° phase difference) to the other would seem to be more rapid than the movement in the remainder of the beat-cycle. The quantitative agreement is however, not satisfactory.

The theory is further confirmed by the fact that with a rapid beat-frequency (more rapid than one per second) not only do the secondary maxima disappear, but the localization in the 180°-phase-difference region disappears also.

Anatomical Considerations.

The query arises, does the structure of the ear permit of the assumptions as made? It would seem that no serious objection arises if we assume the instrument A to be the organs of Corti in the ductus cochlearis, and the instrument B to be the similar organs in the saccule or the utricle. These organs in the cochlea and the saccule and utricle, have a common origin. Herrick[1] states that some physiologists have thought that cochlear and vestibular nerve systems are not wholly distinct and that the sense organs in the saccule may also function as a sound perceptor. This uncertainty concerning the function of the organs in the utricle and saccule on the part of the physiologists does not serve as an objection to our theory. Indeed, the fact that the complex phenomena herein described can be accounted for by the assumption of the existence of the B instruments ought to assist in deciding this uncertainty. The fact that there is a great difference in the central connections of the cochlear and vestibular nerves is in harmony with our contention that the instruments B give sensations which do not completely blend and which do not produce external localization.

But are the vestibular and cochlear organs mechanically different? Obviously they are, for the latter are placed upon a membrane, and are apparently excited by the motions of the fluid in the scala media, of the basilar membrane and of the tectorial membrane. The vestibular organs are located at the points where the membranous labyrinth is fastened to the skull itself. Moreover, the movement of the vestibular fluid must be very small for the vestibular canals are closed. Thus there would seem to be a mechanical difference which corresponds admirably with our assumption that instrument A is excited via the vibration of the membrana tympani and that instrument B is excited by the skull. The only change in our assumption induced by the structure of the ear would

[1] C. J. Herrick, Introduction to Neurology, pp. 201–202.

be the possible insertion of the word "chiefly" after each word "excited" occurring in the preceding sentence.[1]

Restatement of Theory and Agreement with Experiment.

By the assumption of two physical instruments A and B on each side, having a difference in mechanical arrangement, a difference in the method of excitation, a difference in the blending of the sensations of the A pair and B pair of instruments and an independence of sensations produced by the two pairs and the additional assumptions that instruments A, the usual instruments of hearing, can perceive phase differences, and that in the skull conduction to the B instruments there is an additional phase change of 180°, we have a theory that explains fairly satisfactorily the presence of the primary maxima, the secondary maxima, the appreciable length of beat-period requisite for the appearance of the latter, the localization in the entire beat-cycle, the difference in the nature of the localization and in the quality of the sound in the two halves of the beat-cycle, the rapid movement of localization between the secondary maxima, the variation of the separation of the secondary maxima with the fork-frequency and its independence of beat-frequency, the difference in the effects of unequal intensities upon the localization in the two halves of the beat-cycle, and finally the simultaneous disappearance of the secondary maxima and the localization in the region of 180°-phase-difference.

Our theory does not lead us to identify the B instruments, but merely to suggest the possibility of these instruments being located in the utricule and saccule. Anatomical and physiological considerations give evidence in favor of the suggestions.

The theory is, of course, incomplete, but it has herein proved to be very successful and hence promising. There are no conspicuous phenomena connected with binaural beats that are not in accord with the theory, but there are minor points that should receive further consideration. For example, S. P. Thompson[2] has found binaural beats occurring when the tones used have frequencies of almost two to one.

While it is known[3] that the secondary maxima and the localization phenomena occur both when the sounds are presented by tubes which

[1] A paper of importance has just come to the writer's notice. D. Richards (Zeitsch. f. Biol., 1916, 66, 579–609) reports upon experiments with a guinea pig in which there were removed (1) both cochlea, (2) both cochlea and one vestibule and (3) both labyrinths. His conclusion is that sound stimuli may be regarded as adequate stimuli for the vestibular apparatus, but that no conclusion can be drawn as to any sensation produced in this way. This contribution is entirely favorable to the theory presented in this paper.

[2] S. R. Thompson, Phil. Mag., (5), IV, 1877, p. 274.

[3] G. W. Stewart, Phys. Rev., 2d Series, Vol. IX. No. 6, 1917, pp. 502–508.

approach but do not close the opening of the external meatus, yet there may be a difference in the quantitative measurement of 2δ with the ear open and closed. The experiments of the writer have been made almost entirely with binaurals that close the external meatus, and accurately speaking his theory has reference to that experimental condition. Nevertheless he believes it to be equally applicable to the open-ear-binaurals.

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YOUNG'S MODULUS OF DRAWN TUNGSTEN AND ITS VARIATION WITH CHANGE OF TEMPERATURE, INCLUDING A DETERMINATION OF THE COEFFICIENT OF EXPANSION.

BY H. L. DODGE.

THIS paper is the fourth of a series upon the effect of temperature upon the elasticity of wires and deals with tungsten. The method is the same, in general, as that employed in previous work with copper,[1] mild steel,[2] and aluminum[3] wires, but the apparatus has been entirely rebuilt and embodies a number of improvements. In the present form it permits of the measurement of Young's modulus up to a temperature of about 800° C. with external heating and with internal heating to still higher temperatures.

OPEN TUBULAR FURNACE APPROXIMATING A BLACK BODY.

The most important improvement has been in the new furnace, which is entirely different in construction and principle from the one formerly used. The old furnace consisted of a long rectangular box of asbestos board with a glass top. The heating element lay on the bottom. Thus there was a large temperature gradient in the space around the wire and, although every precaution was taken to insure that the thermo-couples should give the temperature of the wire, there was always some error, the possible magnitude of which could be estimated only roughly.

The new furnace is constructed of a series of three coaxial tubes, each thirty inches long. The inner tube is of copper and has an inside diameter of five eighths of an inch and a one eighth-inch wall. Next comes an alundum tube wound with a heating element of nichrome ribbon. Surrounding this is an outside covering of vitribestos. In the top and in the side, 10 cm. from each end, are holes extending through all the tubes. Above the vertical holes are placed two stereopticon lamps for illumination; the wire is viewed through the horizontal holes. All the holes have mica windows.

As the loss of heat from the furnace is very much greater at the ends

[1] PHYS. REV., *2*, 2, 431, 1913.
[2] PHYS. REV., *2*, 5, 373, 1915.
[3] PHYS. REV., *2*, 6, 312, 1915.

than at the center it is necessary to find by trial the best distribution of the heater winding to secure the most uniform temperature throughout the length of the inner tube. The latter, being of heavy copper, smoothes out all local irregularities in temperature.

In a furnace of this kind advantage can be taken of the fact that, except at the ends, the interior of a long tube which is at a uniform temperature is equivalent to a black body.[1] Even though there is an appreciable temperature gradient along the tube it is safe to assume that the temperature over any given cross-section is uniform and the same as that of the wall of the tube at that point. It is also true that every point on a wire suspended in the tube will take up a temperature almost exactly that of the cross-section in which it happens to fall. Therefore, in order to determine the temperature of a certain point on the wire, it is not necessary to place the thermo-couple in contact with the wire but merely to determine the temperature at any point situated in the same cross-section.

However, when the heating is by an electric current in the wire itself this method cannot be followed nor can thermo-couples be applied directly to the wire. It has been found that the most satisfactory method is one depending upon thermal expansion. The coefficient of expansion of the wire having been determined once for all, the same observations of length necessary for the measurement of the modulus also determine the temperature. The expansion coefficient is found by the following method.

Measurement of the Coefficient of Thermal Expansion.

A certain current is passed through the heating element and allowed to flow for a definite time, let us say one hour. During the last few minutes the current is kept very steady by means of a potentiometer. At exactly the end of the hour the electromotive force of a thermo-couple, inserted to the middle of the furnace, is read. A larger current is then passed for a definite time, known to be sufficiently long for the furnace to reach a condition of equilibrium. Readings of the heating current and of the thermo-couple E.M.F. are again taken. This process is repeated until the highest temperature permitted by both furnace and wire is reached.

After the furnace has cooled the thermo-couple is removed and the wire suspended in the furnace. Then exactly the same procedure as before is followed, except that the thermo-couple readings are replaced by measurements of the change of length of the wire. Thus for every

[1] Analogous to the uniform field of a long solenoid.

thermo-couple reading there is a corresponding measurement of the increase of length of the wire, made under identically the same conditions of temperature. From these observations the coefficient of thermal expansion can be readily computed.

The variation of temperature along the tube was investigated in order that the average temperature of the wire over the portion under observation could be found from the temperature measured. When the center of the furnace was at 673° C. at no point between the centers of the windows did the temperature differ from this value by as much as five degrees.

The Tests.

The tests were made upon a piece of drawn tungsten wire secured through the kindness of Dr. A. G. Worthing, of the Nela Research Laboratory. The wire was obtained in December, 1914, and was said to contain approximately 99 per cent. tungsten and one per cent. thorium. It has a diameter of 0.65 mm.; the length under observation was 593.6 mm. The thermal expansion was found to be practically uniform over the temperature range covered, namely 20° C. to 675° C., the coefficient of expansion being .00000456 per degree Centigrade. This value checks exactly, for the temperature range covered, with that of Worthing[1] but

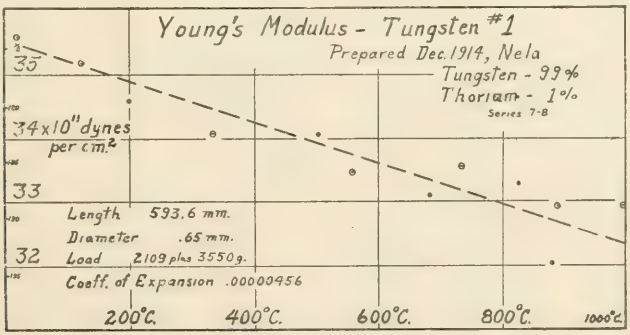

Fig. 1.

Effect of temperature upon the Young's modulus of drawn tungsten.

does not verify that of Langmuir,[2] from which it differs by as much as fifty per cent. Worthing's formula was used in determining temperatures above 675° C.

In measuring the modulus of elasticity the permanent load was 2,109 g., the added load, 3,550 g. The Young's modulus of drawn tungsten was found to be 35.5×10^{11} dynes per cm.² at 20° C. This value is

[1] Jour. Frank. Inst., 181, 857, 1916; Phys. Rev., 2, 10, 638, 1917.

[2] Phys. Rev., 2, 7, 329, 1916.

undoubtedly accurate to within two or perhaps three per cent. This is somewhat lower than the value given by Fink[1] of 42,200 kg. per sq. mm., but it is entirely possible that there is that much difference in the modulus of wires prepared at different places and at different times, for the art of drawing tungsten wires has had a recent and rapid development.

The change of the modulus with increase of temperature was observed up to 1,000° C. at which temperature the oxidation of the wire became very rapid. However it was possible to check back after readings at about 900° C., as shown in Fig. 1. The dots represent observations taken with increasing temperature, the last at 880° C. The cross is the value found immediately after the wire had cooled. The dotted circles represent the next series of readings.

Only two series of readings are shown in the figure. These were preceded by a great deal of preliminary work, necessary to determine the magnitude and general nature of the effect of temperature and to learn what loading should be used. On account of the extremely high value

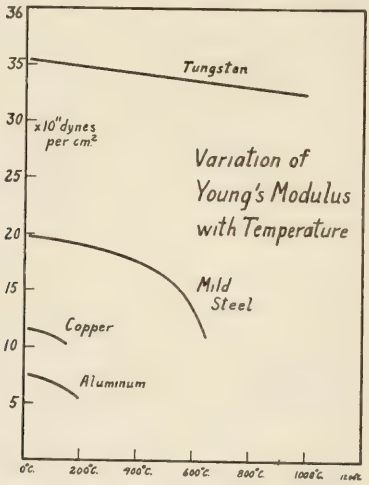

Fig. 2.

Comparative effects of temperature upon the Young's modulus of aluminum, copper, mild steel and tungsten wires.

of the modulus much heavier weights than usual were necessary. Even with a load of over 3.5 kg. the *variation in the stretch over the whole temperature range amounted to but eighteen thousandths of a millimeter.* The actual stretch in thousandths of a millimeter corresponding to the different values of the modulus is indicated at the left edge of Fig. 1.

All of the observations were taken when the heating was by a current

[1] Trans. Am. Electrochem. Soc., 22, 503, 1912.

in the wire itself as it was found that the manner of heating did not affect the results and this method was the more convenient.

A temperature of 1,000° C. is so low in comparison with the melting point of tungsten that one could hardly expect it to show the characteristics of the other metals with lower melting points. However there is nothing in the behavior of tungsten which is not in harmony with the general conclusions already reached regarding the effect of increase of temperature upon elasticity.[1] In Fig. 2 the effects with tungsten and with the three other metals are compared.

Summary.

The Young's modulus of drawn tungsten is 35.5×10^{11} dynes per cm.^{2} at 20° C. The modulus decreases uniformly with increase of temperature up to 1,000° C. at which temperature it is 32.3×10^{11} dynes per cm.^{2}

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[1] Dodge, Phys. Rev., *2*, 6, 316, 1915.

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THE INTENSITY FACTOR IN BINAURAL LOCALIZATION: AN EXTENSION OF WEBER'S LAW[1]

BY G. W. STEWART AND O. HOVDA

University of Iowa

That the ratio of the sound intensities at the two ears is a factor in the localization of the sound source is well known. A number of experimenters[2] have reported investigations in which this ratio was varied and the localization observed, but none of them have attempted quantitative measurements. Previous experiments of the writer[3] on binaural beats indicated that the ratio of intensities when a pure tone is used was not an important[4] factor in binaural localization. This indication was not in harmony with the generally accepted view as to the importance of the intensity ratio, and hence quantitative measurements seemed advisable. This article will be confined to a presentation of these results. The factor of phase difference will not be discussed.

The Method.—The experimental procedure was based upon the belief that a knowledge of the importance of the intensity ratio as a factor in binaural localization should be sought, in the first instance, by the use of simple tones. The results of this paper are for a single frequency, 256 d. v., and the conclusions drawn would furnish only the first step in a complete investigation.

The Apparatus.—The frequency was selected by the circumstance that we already had in our possession a sensitive

[1] From the Physical Laboratory of the University of Iowa. We wish to acknowledge our indebtedness to Mr. Eugene Berry for his assistance as observer.

[2] Tarchanoff, *St. Petersburger med. Wochenschrift*, 1878, No. 43, 353–354. Steinhauser, *Phil. Mag.*, Ser. 5, 1879, 7, 261–274. Urbantschitsch, *Pflüger's Archiv*, 1881, *24*, p. 579. Kessel, *Arch. f. ohrenheilk.*, XVIII., 1882, p. 120. Matsumoto, Studies from the Yale Psychological Lab., 1897, No. 5. Stenger, *Zeitsch. f. Ohrenheilk.* XLVIII., p. 219. Rayleigh, *Phil. Mag.*, 1907, 13, p. 217. Ferree and Collins, *Am. J. of Psychology*, 1911, 22, p. 250. Numerous other writers refer to the subject.

[3] Stewart, *Phys. Rev.*, 1917, 9, 502–528.

[4] Rayleigh, *loc. cit.*, shows that it cannot be an important factor for 128 d. v.

Rayleigh disc[1] made and adjusted for 256 d. v. per second. The source of sound was a tuning fork of this frequency. Two sounds, one for each ear, were conducted from the fork by glass tubes, rubber tubing and binaurals, which, in a portion of the experiments, were inserted in the ears.

In order to obtain as pure a tone as possible and yet to maintain the vibration of the fork by electrical means, the common device of two forks in tandem, as shown in Fig. 1,

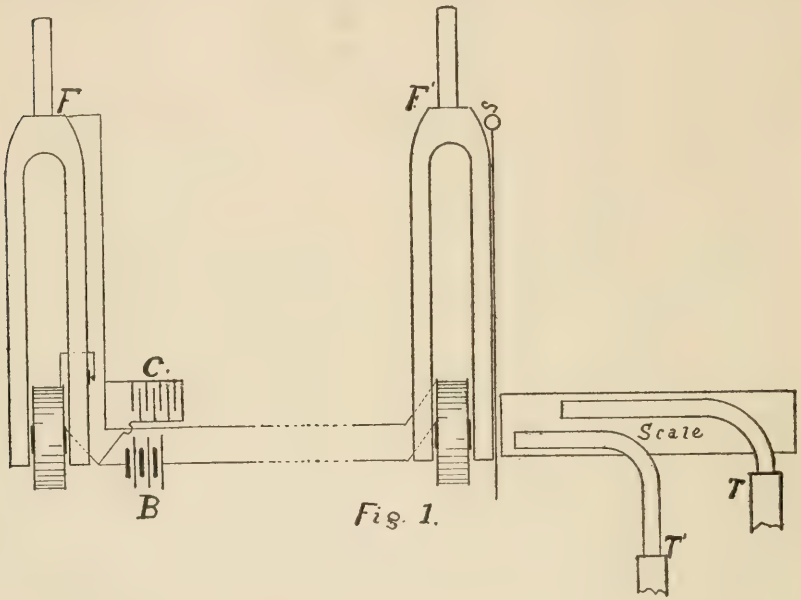

Fig. 1

was used. The auxiliary fork, F, carried the electrical contact and the circuit was completed by the battery and two magnet coils, one for each fork. Two glass tubes were placed in the plane of vibration at the side of one prong of the fork, F', used as the sound source. One of the tubes was stationary and the distance of the other from the fork could be altered. Thus the flow of energy into one tube was modified at will and the relative intensities made any selected value within the range of the apparatus. The screen, S, could be inserted

[1] This disc is described in an article by Stewart and Stiles, *Phys. Rev.*, N. S., 1913, 1, p. 311.

and removed from the space between the fork and the end of the glass tubes. The terminus of the glass tube, T', was permanently fixed at a distance of 1.02 cm. from the fork and the position of the terminus of the other glass tube was varied from 0.20 cm. to 2.20 cm. from the fork. The sound was led from the glass tubes to the binaurals through approximately 3 meters of rubber tubing in each branch. The relative intensities or fluxes of energy in the binaurals were tested by the Rayleigh disc. Into the open end of the disc was inserted a stationary glass tube which will be called the 'disc tube.' Comparative measurements of the flow of energy through either binaural could be tested by connecting the binaural to the disc tube by a short rubber tube. The deflections of the disc were small enough to be considered as proportional to the intensities.

Binaurals constructed in order to avoid contact in the external meatus are shown in Fig. 2. *B* is a piece of thin

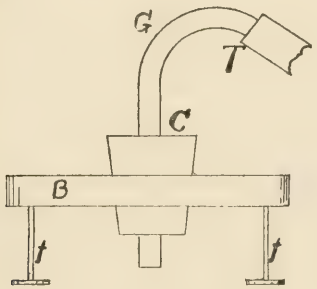

FIG. 2

triangular board, carrying at the corners three feet, *f*, which rest upon the skull. The sound enters through the rubber tubing, *T*, and passes through the glass tube, *G*, the end of which can, by adjustment in the cork, *C*, be placed near the opening of the ear. The real advantage of the binaurals is not that they completely isolate the skull, but that they present the sound to the ears without closing the external meatus. Nevertheless, the sound conveyed by the feet to the skull is vanishingly small and without influence.

In a part of the experiments the binaural observer was in the same room with the forks, but when the non-contact

binaurals were used the observer was in an adjacent room, in which the sound from the forks was sub-liminal. In front of the binaural observer was placed a chart marked with radial lines representing angular displacements from the median plane. The second experimenter was stationed at the sound source, and had the duty of varying the intensity of sound received at one ear by altering the distance of the adjustable tube at the fork. By resistance adjustment he also kept the amplitude of the fork approximately constant, this being a precaution to avoid any large variations in the flux of energy from the fork.

In each set of experiments fifty different values of intensity-ratio were used. In order that these values might be properly distributed in a given set the desirable displacements of the tube were chosen and these arranged 'at random.' The binaural observer was thus wholly without any information that would influence him in favor of a certain angular displacement.

Several assumptions and approximations were made in the arrangement of apparatus and the interpretation of results. They are as follows:

1. It was assumed that, for a given adjustment, the intensity-ratio of the stimuli at the two ears is the same as the ratio of intensities produced at the Rayleigh disc. The assumption seems to be entirely warranted.

2. It was assumed that the vibrations produced at the ears were alike in phase. This is only approximately true. The adjustable tube at the fork was displaced not to exceed one centimeter from its mean position. Thus the maximum change in phase produced by the altered length of path was less than 3°. Since former experiments with change of phase[1] indicates that the angular displacement in localization by a difference of phase is of the same order as the phase difference itself, one can be confident that the error of 3° does not modify the conclusions of our experiments.

3. In the interpretations of the results it is assumed that our method of producing intensities at the ear is physically

[1] G. W. Stewart, *Phys. Rev.*, 1917, 9, pp. 502–508.

equivalent to that of moving a source of sound about the head, with the important difference, however, that in the method used the phase difference is negligible. This assumption disregards any effect produced by the impinging of waves upon the skull, which would occur in the case without binaurals. But experiments with the external meatus carefully

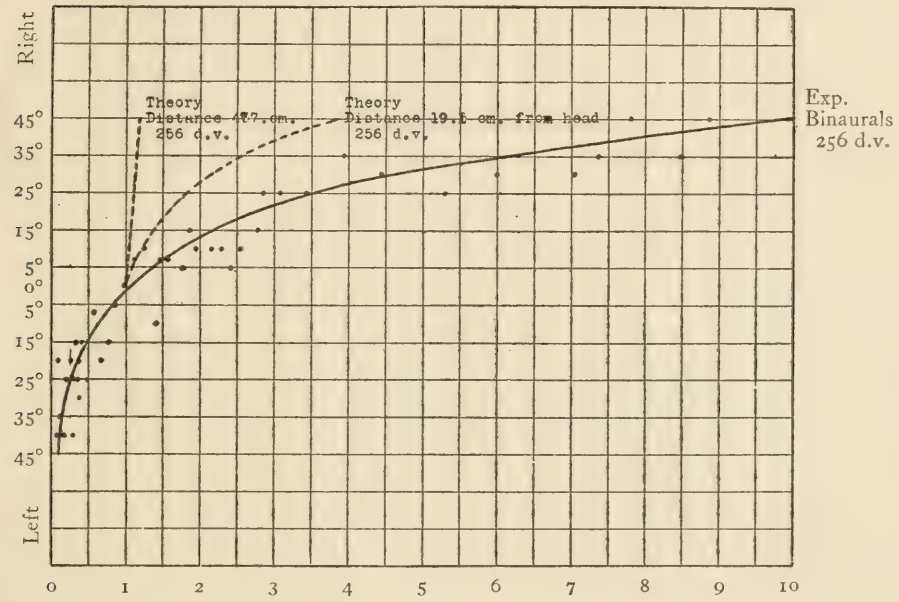

FIG. 3. Relative intensities.

stopped proved that the intensity of sound conveyed from the skull to the drum-skin under such circumstances was subliminal.

The Results.—The results first to be mentioned are those made with ordinary stethescope binaurals. The screen at the fork was opened for one or two seconds only, the operation being repeated with silences of perhaps one second each until the binaural observer indicated in some manner that he had arrived at an opinion concerning the apparent angular displacement from the median plane. In Fig. 3 the observations of one set are shown by dots and the mean results by the full-line curve.

The ordinates represent the apparent angular displacements from the median plane of the source of the blended sound and the abscissæ the ratio of intensities of the stimuli at the ears. The values last named we obtained from a knowledge of the distances of the movable tube as shown in the scale, Fig. 1, and a calibration curve taken with the aid of the Rayleigh disc. This curve was obtained by repeated measurements of the relative intensities with various distances on the scale, Fig. 1. The calibration measurements and curve are omitted, inasmuch as they will not be discussed in any manner. The error in the knowledge of any actual intensity-ratio does not exceed five per cent. and is usually much less.

The most striking result given by Fig. 3 is the relatively large change in intensity necessary to produce displacement in localization. The intensity at one ear had to be ten times that at the other in order to produce a displacement of 45°. This is surprising, for an intensity-ratio as great as 10 to 1 at the ears never occurs in our common experiences. By way of comparison, let us ascertain what values of intensity-ratio would be actually obtained by displacing a source of sound 45° from the median plane. For such values we are dependent upon a theoretical investigation,[1] in which were found the relative values of intensities at various points of a rigid sphere 60 cm. in circumference, with the source at several distances, r. In order to utilize the results of the theoretical investigation, we may select two points diametrically opposite on the sphere and assume the source to be carried about the sphere in a plane containing this diameter. From Fig. 2 of the reference just cited, it is possible to get the values of the intensities at these diametrically opposite points for a source of a wave-length 120 cm. (or, nearly enough), the frequency here used at distances of 477 cm. and 19.1 cm. from the sphere. It is a simple matter then to take the ratio between these intensities for any angular displacement of the rigid sphere corresponding to the apparent angular displacement observed and shown in Fig. 3. The dotted line curves in

[1] Stewart, *Phys. Rev.*, 33, p. 473.

Fig. 3 show these computed values of intensity-ratio and thus can be assumed to represent approximately the ratios of intensities at the ears with a source of sound 256 d. v., at distances of 477 cm. and 19.1 cm. It is apparent that with a displacement of 45°, the change in the theoretical ratio of intensities is 20 per cent. and 300 per cent. for the distances

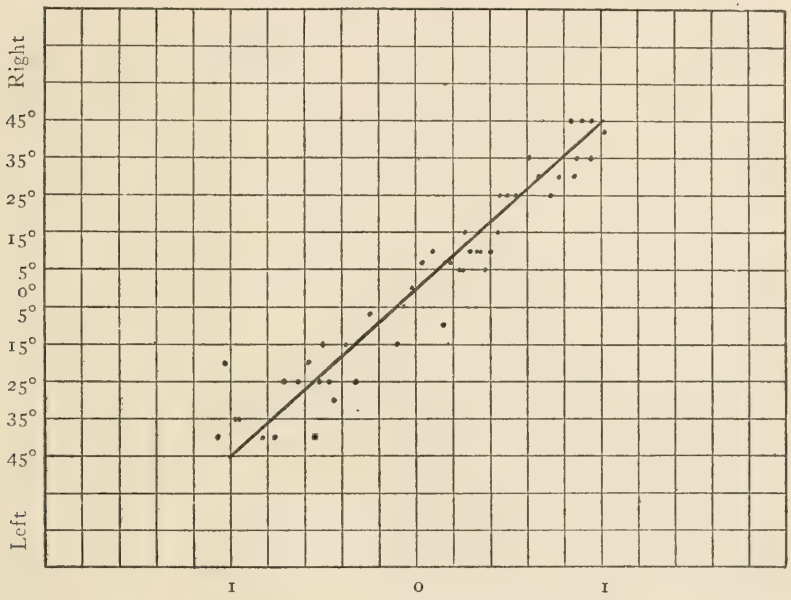

Fig. 4. Log of relative intensities.

from the center of the head of 477 cm. and 19.1 cm. respectively. But in our present experiment the change in the ratio required to produce apparently this same displacement is 1,000 per cent. It is perhaps fair to compare our present experimental intensity-ratio with the theoretical values obtained when the source of the same frequency is placed at the not unusual distance of 477 cm. We have then to compare an experimental change of intensity-ratio of 1,000 per cent. with the theoretical of 20 per cent. The comparison shows that the localization of a simple tone of 256 d. v. cannot be due to the intensity-ratio, for a change in ratio of 20 per cent. as shown by these experiments, would produce an ap-

parent displacement of less than 4°. But our experiments deal with intensity changes *only*. Hence our conclusion is that the intensity factor alone is of relatively small importance with a simple sound source. The work 'relatively' is used advisedly for it is well known that the difference of phase, when that alone occurs, is a very important factor.[1]

If the experimental results presented in Fig. 3 are plotted with the logarithms of relative intensities as abscissæ, the results are as shown in the accompanying Fig. 4.

Clearly the mean is best represented by a straight line. This leads at once to the interesting conclusion that the angular displacement, θ, can be expressed in terms of the right and left intensities, I_r and I_e respectively, in the following equation.

$$\theta = \mathrm{K} \log \frac{\mathrm{I}_r}{\mathrm{I}_e}, \qquad (1)$$

where K is a constant.

Before discussing the significance of this law, a record should be made of all the observations in our experiments. There were three observers, and each took a set of observations similar to that shown in Fig. 4. Also each took a set of observations with the non-contact binaurals as shown in Fig. 2. All the observations amply verified (1), but the slope of the non-contact-binaural-curve was generally slightly different, with one observer less and with another greater. Curves indicating the results obtained by one observer are shown in Fig. 5.

Here the slopes of the three full-line curves are 0.81, 0.91 and 0.75, averaging 0.82. The slope of the dotted line, or the non-contact binaural, is 0.63. The results with the non-contact binaurals show an even greater insensitiveness to changes of intensity at the ears and hence the contact binaurals do not lead to incorrect conclusions because of the closing of the external meatus. From the dotted line in Fig. 5, we find the ratio of intensities required to cause a displacement of 45° is approximately 25 to 1.

The results obtained with the other two observers were

[1] See references given by Stewart, *Phys. Rev.*, Second Series, 1917, 9, p. 502.

similar. The average slopes of the contact binaural curves were 0.96 for one and 0.93 for the other observer.

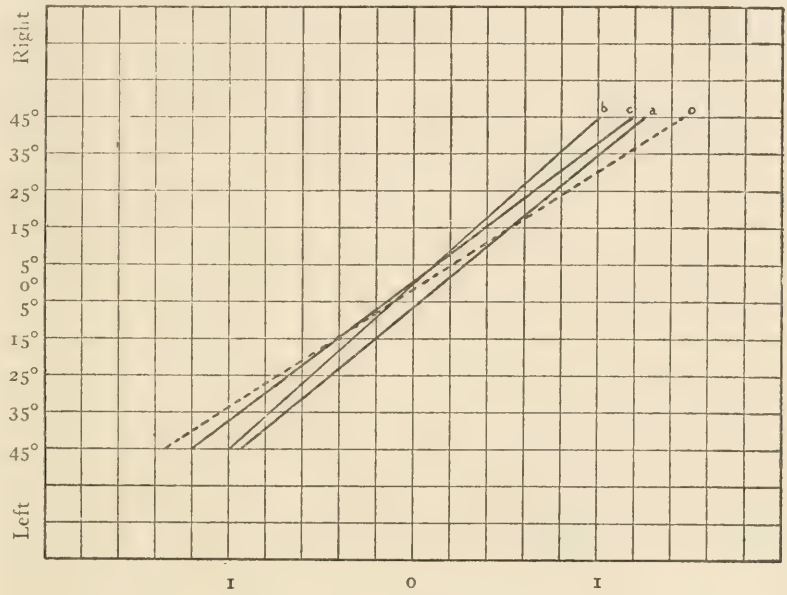

FIG. 5. Log of relative intensities.

In the experiments evidence was found that under certain circumstances the results might be best represented by two straight lines meeting at approximately 0° on the chart. These unusual results will not be discussed in this paper for it is believed that the results already shown represent the normal case.

Equation (1) reminds the reader of Weber's law. In its integrated form it states that the response is proportional to the logarithm of the stimulus. Its apparent truth is found in experiments on vision, audition, pressure, smell, and kinæsthetic complexes, indicates that the law is one of the nervous system and is not dependent upon the nature of the peripheral organs. The discovery expressed in (1) is, however, an interesting and perhaps an important extension of Weber's law, for here we find that the response of the individual as indicated by the constructive angular displacement

is proportional to the logarithm not of one stimulus, but of the ratio of the two stimuli. The above results open up many questions of interest both to the physicist and to the psychologist. The experimental work must be extended until the importance of intensity and phase relations, both singly and together, both with simple and complex tones and for all frequencies is clearly understood.

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THE VARIATION IN THE BLACKENING OF A PHOTOGRAPHIC PLATE WITH TIME OF EXPOSURE, TOTAL ENERGY REMAINING CONSTANT.[1]

BY P. S. HELMICK.

FOR many years following the discovery of photo-chemical action, it was believed that if the product of the intensity of light producing the exposure and the time of exposure were constant, the resulting photo-chemical effect would be constant. R. Bunsen and H. Roscoe[2] expressed this idea as early as 1862. Subsequently Abney,[3] Miethe,[4] Eder,[5] Michalke and Schiener, Schwarzschild,[6] Lemon,[7] Kron,[8] and others showed variations from this so-called "Reciprocity Law," but directed their attention to the determination of relations between intensity and time which would give constant blackening, rather than finding the variation in the blackening with time, with total energy remaining constant. Abney[9] and Kron[10] seem to have made the only progress in this last-named problem. Both claim that when blackening is plotted against time of exposure, with total energy remaining constant, that the resulting curve will show a maximum; but neither investigator directly obtains this curve.

It has been the purpose of the writer to investigate Abney's and Kron's conclusion, and actually obtain the curve showing the maximum, if it existed. By exposing the plate to different rates of flow of constant light energies, it was believed that some additional knowledge of the physical mechanism in a light-sensitive plate might be gained.

[1] A paper read before the American Physical Society, December, 1916.

[2] Ann. der Phys., 117, 538; 1862.

[3] "Chemical Action and Exposure," Phot. Journ., Oct., 1893; "The Failure of a Photographic Law with Intense Light," J. C. C., 8, 46.

[4] Inaug. Diss. Gottingen., 1899.

[5] Handbuch, Band 2, Jahrbuch, 1899, 457.

[6] Phot. Corr., 1899, 171; Beitrage zur Phot. Photem. d. Gestirne; Astrophys. Journ., 11, 89, 1900.

[7] Astrophys. Journ., 39, 204, 1914.

[8] Ann. der Phys., 41, 755, 1913.

[9] Treatise on Photography, 395, 1901.

[10] Loc. cit.

APPARATUS AND METHOD.

The plates used were coated on special plate glass, and the variations of density due to unevenness of coating were of the order of 1 per cent. The following emulsions were used: Seed 23; Seed 27 G. E.; and Seed Graflex. The sensitiveness of these plates in the camera was roughly 1, 3, and 9.

Two sources of light were used: the integral light from a 4 volt carbon lamp, and green light of wave-length 545 $\mu\mu$ transmitted through a Hilger monochromatic illuminator from a 32 C.P. coil filament tungsten lamp. The plates were exposed in a light-tight box 350 cm. long. The intensity of the light was varied by altering the distance between the plate and the light, and the value of the intensity for any distance was computed by the "inverse square law," as modified by Hyde[11] for finite sources.

Exposures greater than one second were made by a sliding shutter operated by hand, with the aid of a telephone receiver clicking seconds. Shorter exposures were made by a modification of an apparatus used by Wood.[12] Electric contacts attached to a large sector disk rotating at a constant predetermined speed operate an auxiliary sliding shutter, permitting the shutter to be opened just before the revolving sector disk reaches the point where it allows the plate to be exposed, and permanently closing the shutter immediately after the sector disk exposes the plate. The minimum exposure, uniform to 91 per cent.,[13] which this particular apparatus could give with a sector speed of 10 r.p.s. was 1/37,600 second.

Plates were developed for constant time and practically constant temperature in a developer compounded after Brush's formula.[14] Densities were measured in a modification of Lemon's spectrophotometer[15] in which the prism is replaced by two mirrors inclined to one another so as to reflect two beams of light into the observing telescope. One beam of light, reduced in intensity by the interposed plate whose density it was desired to measure, was matched with another beam whose intensity was regulated by the rotation of a nicol prism. The density of the plate in terms of the angle of rotation of the nicol is given by the expression $\text{Log}_{10} \text{Sec}^2 \theta$.[16]

[11] Bull. Bur. Stands., 3, 81, 1907.

[12] Phil. Mag., 6, 577, 1903.

[13] Traite Encyc. de Phot., 1, 436.

[14] PHYS. REV., 31, 243, 1910.

[15] Loc. cit.

[16] A table of $\text{Log}_{10} \text{Sec}^2 \theta$, with differences to 0°.01 has been prepared by the writer, and may be obtained on request from the librarian of the State University of Iowa, Iowa City, Iowa.

Results.

The three different brands of plates were exposed to the integral, or "white" light of the carbon filament lamp, and to the monochromatic green light of wave-length 545 $\mu\mu$, and plotting density and time, a definite

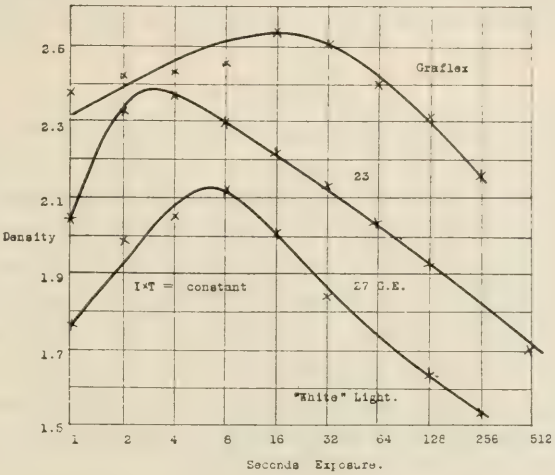

Fig. 1.

maximum density as time varied but with intensity times time constant, was obtained in each case. A few of the curves obtained are shown in the two figures, and are characteristic of all the curves obtained. The

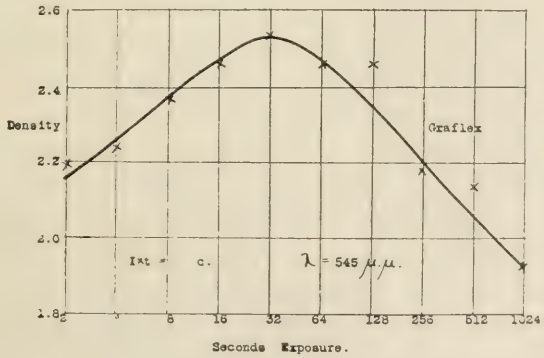

Fig. 2

curves show that the blackening of a plate is dependent upon the rate of flow of energy, with total energy constant; and that for each brand of plate and quantity of total energy there is a maximum blackening given by a certain rate of flow of energy.

The relative heights of the curves in Fig. 1 are not important, for no fog strip was taken, and the temperature varied a trifle, but the relation between the time of exposure to produce maximum blackening and the speed of the plate seems significant, for with the same value of total energy upon each plate, the positions of the maxima vary as the speeds of the plates. By using this rule, the writer was able to shift the maxima of the curves at will.

SUMMARY.

1. An accurate electric shutter has been designed for photographic exposures.

2. A simple density-determining apparatus has been described.

3. Plates of three different speeds have been exposed to white and to green light. The rate of flow of energy was varied, but the total energy the plate received was kept constant. In every case there was a maximum blackening, and the time of exposure to produce maximum blackening varied as the speed of the plate.

In conclusion, it is a pleasure to acknowledge the encouragement received from the staff of the department of physics of the State University, and particularly from Professor H. L. Dodge.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA.

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WAVE-LENGTHS OF THE TUNGSTEN X-RAY SPECTRUM.

BY ELMER DERSHEM.

INTRODUCTION.

SINCE the X-ray spectra of practically all the available elements had been studied by one investigator or another with results which did not very closely agree and which in general comprised only a few of the principal or most prominent lines, it seemed wise to begin the present investigation with a view to determining more completely and accurately than heretofore the number of lines and their wave-lengths in the spectrum of at least one element. The element most easily tested and the one whose spectrum would be of the greatest value in the X-ray analysis of crystals was tungsten on account of its use as the anticathode of the Coolidge tube, the only type of tube which could be used during the long intervals of time necessary to secure spectral photographs, if the conditions required for the greatest resolving power and the greatest accuracy of measurement were complied with.

The photographic method was chosen for this work in preference to an ionization chamber and electrometer because in the latter method the intensity of the reflected beam must be great enough to give a continuous effect on the electrometer while the photographic plate gives a summation of the intensity of the reflected beam over a time that may be made so very much longer that weak lines have an opportunity to appear.

We shall now consider the factors affecting the accuracy of measurement and resolving power of an instrument using a crystal as a diffraction grating for X-rays. Resolving power is, as usual, defined as the ratio of a wave-length to the smallest difference which may exist between this and a neighboring wave-length and yet have the instrument show that the two waves are separate and not identical. A consideration of these factors will then show that the conditions for the best resolving power are those which lead to a decrease in intensity and would make impossible the securing of sufficient intensity to affect an electrometer under the necessary conditions of a narrow source, great distance from the crystal to the detector and a thin crystal which means less intensity because there are fewer reflecting planes. The theory will also show that the position of the central maximum of the reflected beam is not the true criterion by which the wave-length must be determined but it is instead

the outer edge, which when corrections are made for the width of the source, gives the true measurement. The impossibility of measuring anything other than the central maximum with an ionization chamber eliminates this as a possible accurate method and leaves the photographic plate as the only recourse.

RESOLVING POWER OF A CRYSTAL USED AS A DIFFRACTION GRATING FOR X-RAYS.

In this discussion the assumption will be made that the slit, or source, is the same distance from the crystal as is the photographic plate. In this case, as shown by Bragg,[1] the amount of surface of the crystal exposed to the X-rays makes no difference in the sharpness of the lines since the same wave-length is always reflected to the same point on the plate. This will not be true if the atomic planes are not parallel. In reality the cleavage surfaces of crystals are quite noticeably warped and it is desirable to limit the surface of the crystal exposed to the rays by means of a narrow slit between lead blocks placed close to the crystal even though it does cause a decrease in intensity. It will also be assumed that the crystal is thin enough that the rays may penetrate entirely through the crystal and be reflected from the planes on the back side and again traversing the crystal to reach the photographic plate.

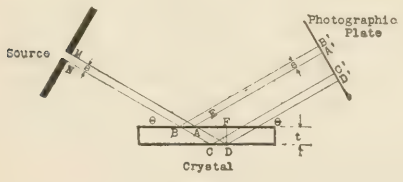

Fig. 1.

With these assumptions as to conditions which may be easily obtained in practice, the question to be determined is, What difference of wave-length is necessary that it may be possible to separate two waves of nearly the same length?

Let the source be a slit of width s at a distance r from the crystal, Fig. 1. Assume that the crystal is in a position to reflect some particular wave-length where $n\lambda = 2d \sin \theta$, in which n is the order of the spectrum, λ the wave-length, d the grating constant or distance between the atomic planes and θ the angle between the incident rays and the crystal surface. Then a ray coming from the side M of the slit may be reflected at A to A' on the photographic plate and a ray from the side N must strike the crystal at the same angle and consequently be reflected at the point B to the point B'. It is easily seen that the reflected rays AA' and BB' are at the same distance apart as the incident rays MA and NB. Hence due to the slit alone a single wave-length would cause a line on the photographic plate the same width as the slit.

[1] Bragg and Bragg, X-Rays and Crystal Structure, G. Bell and Sons, London, 1915.

Considering next the question of the variation of the width of image with the thickness of the crystal, let DE be drawn perpendicular to AA'. Then DE is the width of the reflected beam due to the penetration into the crystal. Let $DF = t$ be the thickness of the crystal. Then $t = AD \sin\theta$ and

$$AD = \frac{DE}{\sin 2\theta}.$$

Then by substitution

$$t = \frac{DE \sin\theta}{\sin 2\theta} = \frac{DE \sin\theta}{2 \sin\theta \cos\theta} = \frac{DE}{2 \cos\theta}.$$

Whence $DE = 2t \cos\theta$.

Since DE is the width of beam due to penetration into the crystal the total width of beam is $s + 2t \cos\theta$, in which s is the width of the slit, or source, t the thickness of the crystal and θ the angle which the incident ray makes with the crystal.

Then $s + 2t \cos\theta$ is the width of the line on the photographic plate. In order to resolve two lines of nearly the same wave-length it is necessary that their images on the plate should not overlap or, in other words, that the centers of their images must be further apart than the width of beam, $s + 2t \cos\theta$.

Assume two wave-lengths, λ and $\lambda + \Delta\lambda$. To find how small $\Delta\lambda$ may be and these wave-lengths still be clearly resolved on the plate. Using the formula $n\lambda = 2d \sin\theta$ let λ take on a small increment $\Delta\lambda$ and θ the corresponding increment $\Delta\theta$. Then by differentiation we have $n\Delta\lambda = 2d \cos\theta\Delta\theta$. This is justified in practice by the fact that $\Delta\theta$ is small in comparison to θ.

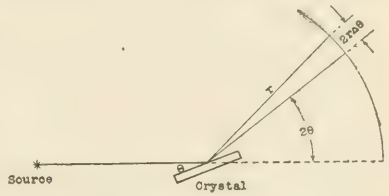

Fig. 2.

According to the above if the crystal is in a position to reflect a wave of length λ it must rotate through an angle $\Delta\theta$ in order to reflect a wave of length $\lambda + \Delta\lambda$ and since the reflected ray rotates twice as fast as the crystal the reflected ray must rotate through the angle $2\Delta\theta$. (See Fig. 2.)

If the distance of the crystal from the plate is r then the displacement of the beam along the plate when the reflecting angle is changed from θ to $\theta + \Delta\theta$ is $2r\Delta\theta$. In order that rays reflected at these angles be separated it is necessary that this distance, $2r\Delta\theta$ be greater than the width of beam $s + 2t \cos\theta$.

$$2r\Delta\theta > s + 2t \cos\theta.$$

But

$$\Delta\theta = \frac{n\Delta\lambda}{2d\cos\theta}$$

and by substitution

$$\frac{2nr\Delta\lambda}{2d\cos\theta} > s + 2t\cos\theta$$

$$\Delta\lambda > \frac{d\cos\theta}{nr}(s + 2t\cos\theta).$$

$\Delta\lambda$ is then the smallest difference between the lengths of two waves that is permissible if the images due to these waves are to be separated on the plate. However the images must be separated by a slightly greater distance in order to leave a clear space between them. Just how much space is necessary for this is not a mathematical problem but a question which must be answered by experience. Probably but little need be added to $\Delta\lambda$ on this account. Neglecting for the time being the question of the necessary space between lines it may be of interest to determine the resolving power under the best conditions that were obtained with the apparatus used in the present work. For example, taking the line in the central part of the L spectrum having a wave-length of 1.241×10^{-8} cm. the experimental values of the quantities contained in the above formula were:

$$s = 0.032 \text{ cm.},$$

$$t = 0.019 \text{ cm.},$$

$$d = 2.814 \times 10^{-8} \text{ cm.},$$

$$r = 62 \text{ cm.},$$

$$\cos\theta = 0.977,$$

$$n = 1.$$

Substituting in the inequality

$$\frac{d\cos\theta}{nr}(s + 2t\cos\theta)$$

the above values of the quantities gives

$$\Delta\lambda > 0.00375 \times 10^{-8} \text{ cm.}$$

Since resolving power is defined as $\lambda/\Delta\lambda$ and for this case λ is 1.241×10^{-8} cm. we have

$$\frac{\lambda}{\Delta\lambda} < \frac{1.241}{.00375},$$

$$\frac{\lambda}{\Delta\lambda} < 331.$$

The resolving power in this case was less than 331, although the experimental values of the width of slit, thickness of crystal and distance to the plate were so chosen as to give the greatest possible resolving power consistent with the necessary requirement of retaining sufficient intensity in the reflected beam to affect the photographic plate in an exposure of a reasonable duration.

It is apparent that the ways in which the resolving power may be increased are to use a higher order than the first, to narrow the source, to decrease the thickness of the crystal and to increase the distance between the crystal and the plate. To do any one of these things tends to decrease the intensity and make necessary a longer exposure and this is not altogether desirable, as it gives the latent image an opportunity to spread and blurr the image and also increases the liability to fogging of the plate due to stray radiation. An increase of distance from crystal to plate decreases the intensity by absorption in the air and this may be a factor of considerable importance in working with the longer wave-lengths. Therefore at present it would not seem possible to so greatly increase the resolving power of a crystal used as a diffraction grating for X-rays as to make it at all comparable to the resolving powers of the grating or echelon used for ordinary light.

From this theory it may be seen, by reference to Fig. 1, that the true angle of reflection must be determined by measuring the position of the outer edge or most deviated portion of the spectral line and subtracting one half of the width of the source from this. This will eliminate any error of measurement due to penetration into the crystal but the crystal must be thin if two nearly equal wave-lengths are to be separated.

Methods of Applying the Theories Concerning Resolving Power.

Since it was the object in this work to make as accurate measurements of the wave-lengths as possible the apparatus and methods of using it will be described somewhat in detail.

The previous theory requiring the use of a thin crystal, the following method of securing and mounting one was adopted. A crystal of rock salt having a perfect cleavage face of about one square centimeter area was chosen and this was fastened face down onto a glass surface by the use of a wax especially prepared for the purpose by mixing Canada balsam and hard sealing wax in such proportions as would give a wax that was hard and tough at ordinary temperatures but which became a thin liquid when slightly heated. After the crystal was firmly cemented to the glass by pressing the two together while warm with a small quantity of wax between and allowing them to cool, the crystal was ground away

until a thickness of not more than 0.019 cm. remained. It was found by experience that attempts to make the crystal thinner than this resulted in causing the crystal to crack and become useless.

The measurements of the position of the lines on the photographic plates were made with a Societa Genevoise dividing engine which was guaranteed by the makers to be accurate to 0.01 mm. in a total length of 40 cm.

To check against possible variations in the pitch of the screw the plates were measured a number of times and each time the setting was changed so that the measurement would be made by a different part of the screw. However the principal object of repeating the measurements was to compensate for the errors of setting by securing a number of readings and averaging the results.

A number of different methods of securing accurate settings of the dividing engine were tried and the one giving the most consistent results was the following. An achromatic combination lens of $1\frac{1}{2}$ inches diameter was placed in a tube 22 inches long. Two parallel hairs were placed at one end of the tube and brought very close to the photographic plate so that the parallel hairs and the spectral line on the plate should be practically in one conjugate focal plane of the instrument at the same time. The spectral line and the parallel hairs were then viewed through a peep hole at the other end of the tube which was near the other conjugate focus of the lens. Owing to the great length of the tube as compared to the distance between the parallel hairs and the photographic plate there was very little parallax and owing to the large diameter of the lens the field of view was large enough to avoid to a considerable extent the loss of contrast that comes from magnifying a small section of surface which shades gradually from one portion to another. It is this difficulty that makes it impossible to use the ordinary microscope having a small objective. To secure proper illumination the apparatus was placed so that the observer looked through the plate toward a clear sky.

Whenever two objects are very close together they appear to blend into one, especially if the edges are not sharp and clearly defined. Owing to this effect as the photographic line approaches the parallel hairs of the microscope it blends with them while not really coinciding with them. To avoid as far as possible, the inaccuracies due to this effect, small dots were made with the point of a needle as nearly as possible along the outer edge of the line and it was then possible while the line was in the field of view of the microscope and yet not too close to the parallel hairs to choose the particular dot which most nearly denoted the position of the edge of the line and then take the measurement when this dot came exactly between the parallel hairs.

DESCRIPTION OF APPARATUS USED IN SECURING THE X-RAY SPECTRUM.

The apparatus used in this work can perhaps best be described by referring to the isometric drawing of the framework, Fig. 3.

The mechanism was enclosed in a box lined with sheet lead $\frac{1}{8}$ inch thick in order to cut out stray radiation, but for simplicity this is not shown in the drawing. The crystal was mounted on the rotating axis A which was fitted with adjustable bearings such that this axis could be made truly vertical with respect to the horizontal plane of the instrument. Between the source and the crystal, as close as possible to the latter, a vertical lead plate $\frac{1}{2}$ inch thick was placed. This is not shown in the drawing. The area of crystal surface upon which the X-rays might strike was limited by a slot 3 mm. wide cut through the center of this plate.

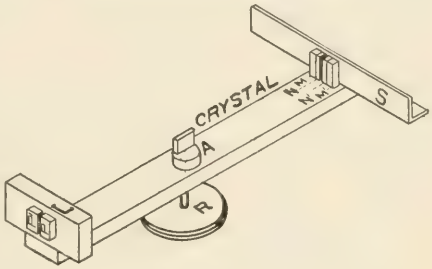

Fig. 3.

One end of the framework of cast iron and steel carried the block of lead L which was about 2 inches thick and of sufficient area to subtend a solid angle at the anticathode of the X-ray tube greater than that subtended by the photographic plate and in this way served to protect the plate from the direct radiation of the tube. The previously mentioned lead-lined box enclosing the apparatus served to protect the plate from the radiation reflected from the walls of the room. A slot about 3/16 inches wide was cut through the center of this block of lead and this slot was covered by the two lead plates or jaws P and P' which had their inner surfaces plane polished and which could be set at any distance apart by means of gauges placed between their upper and lower edges. The slot or space between these two surfaces could then be considered as the source of the X-rays, since it was sufficiently close to the focal spot of the target that this spot subtended a larger angle at the slit than did the crystal, the latter being comparatively far away.

The other end of the framework carried a bar of angle steel, the vertical surface S of which was planed true and then set accurately at right angles to the line joining the center of the source and the center of the rotating axis on which the crystal was mounted. The photographic plate was placed in a light-proof envelope and clamped tightly to this surface and since the distance of the surface from the center of rotation of the crystal could be accurately determined by means of a bar of adjustable length which could later be measured on the dividing engine, it was

possible to determine the distance of the film from the center of rotation by subtracting the thickness of the plate and the paper back of the plate from the measured length of the bar.

The mechanism for holding the crystal is shown in Fig. 4. One side of the shaft A was plane surfaced as was also the block of brass F and these could be firmly clamped together by the two screws H and K. These surfaces could then be separated and placed together at will, always fitting together in the same position. The block F carried the block E attached to it by three screws in such a way that the surface BC could be adjusted to the desired plane and then locked there by the pressure of the screw I. With the shaft set in its bearings the upper and lower parts of the surface BC were adjusted until when viewed through a microscope both the upper and lower edges remained in the axis of rotation as the shaft was rotated. Then this surface BC would contain the axis of rotation and by pressing a crystal surface against this face plate and waxing firmly from behind, the crystal surface would also contain the axis of rotation. The face plate could then be removed by taking out the screws H and K and the crystal would be left properly mounted.

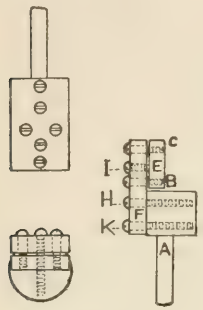

Fig. 4.

The axis A was made perpendicular to the framework by first placing a piece of silvered glass in the position of the crystal and adjusting the bearings until the image of a straight horizontal line drawn along the middle of the surface S was projected back onto the line at all points as the axis was rotated. When these adjustments were made it was assured that the axis of the shaft bearing the crystal was perpendicular to the horizontal plane of the instrument and that whenever a crystal face was placed against the removable face plate its surface would also contain the axis of rotation. The only other adjustment was to set the apparatus as a whole so that the slot between the jaws P and P' was on the straight line joining the focal spot and the axis of rotation of the crystal.

It was necessary to have a precise reference line marked on the photographic plate near the point where the undeviated portion of the X-ray beam would strike in order that a photograph might be taken with the crystal set to reflect toward one side of the apparatus and later one taken on another plate with the crystal turned to reflect to the other side of the center line. From these two plates the mean distance of any particular spectral line from this reference line could be found and having once

determined the position of this reference line with respect to the true center it was possible to determine the true deviation of any wave-length from a photograph taken on one side of the instrument. To check against changes of position the instrument was frequently calibrated by taking photographs on both sides of the center. The reference line was made by allowing part of the portion of the X-ray beam which passed undeviated through the crystal to pass through the narrow slot between the two plane surfaced lead bars N and N' which were soldered to the brass bars M and M' for the purpose of strength and stiffness. These lead surfaces were separated by thin strips of paper between their upper and lower edges and the narrow beam of X-rays that passed through marked a very fine line on the plate.

While the photographs were being taken the crystal was slowly rotated by means of a fine wire which extended from the pulley R, Fig. 3, to a lever which was connected to a float in a tank of water. Water was siphoned into this tank from another tank in which the level was maintained constant and by regulating the rate of flow, the rate of rising of the float, and the rotation of the crystal could be regulated to any value desired.

While taking the photographs of the L radiation the current for the Coolidge tube was supplied by a transformer excited directly from the 110-volt alternating current mains. The transformer stepped the voltage up to a maximum potential of 58,000 volts and the tube rectified its own current, a well-known property of the Coolidge tube provided, as in this case, that the temperature does not become too high.

In order to avoid the necessity of remaining in the room during the long time required for taking the spectral photographs a motor-operated rheostat was placed in the heating circuit of the Coolidge tube and the motor controls were placed in another room. A wattmeter in this room indicated the power input to the transformer and it was possible by regulating the heating current of the tube to secure any power input desired. It was found that when the heating current was such that the power input of the transformer was 240 watts the target remained at a cherry red heat but did not get hot enough to cause damage to the tube. Of this power about 100 watts went to supply the losses in the transformer and the remaining 140 watts represented the power actually used in the tube.

For the K radiations the same method was followed except that the applied maximum potential was raised to 80,000 volts and the current through the heating circuit was set at such a value as to cause the tube to take 140 watts from the transformer as before.

It was found that the power input would remain constant for an hour to within five or ten watts, hence it was possible to work at other things during the long time of exposure required and thus the labor was very much reduced.

EXPERIMENTAL RESULTS FOR THE *L* RADIATIONS.

Some writers on this subject have used the first letters of the alphabet to designate the shorter wave-lengths and others have used these same letters to indicate the longer wave-lengths, while others have used Greek letters. Owing to these confusing methods of nomenclature it has been thought wise to submit the following means of identifying each particular wave-length. The first three significant figures denoting the wave-length in Ångström units are used as subscripts to the Greek letter λ which is usually used to denote a wave-length. If the knowledge of X-ray spectra shall increase to that point where three figures no longer distinguish two neighboring wave-lengths it will be possible to use four or more figures.

In the experimental work a number of photographs were taken using different distances from the crystal to the plate, always keeping the distance from the source to the crystal as nearly as possible equal to this distance. The method of procedure is shown by the following example. Plate No. 104 was placed so as to register the center line and the spectrum on the left side. Later Plate No. 105 was similarly placed on the right side, each being given an exposure of more than twenty-four hours. When measured on the dividing engine the distance of the most deviated side of the spectral line $\lambda_{1.27}$ from the central reference line was found to be 29.99^3 cm. to the left on Plate No. 104 and 30.03^9 cm. to the right on Plate No. 105. The reference line was therefore one half of the difference or 0.023 cm. to the left of the true center. This correction could then readily be applied to photographs taken later on only one side of the apparatus. The deviation of the outer edge of this spectral line was therefore 30.01^6 cm. and since the slit width was 0.032 cm. subtracting one half of this according to the previous theory gives the true deviation of the line to be 30.00^0 cm. The distance from the axis of rotation of the crystal to the plateholder was 61.10^0 cm., from which must be subtracted the thickness of the plate 0.260 cm., also the thickness of the paper envelope enclosing it, which was 0.013 cm., giving 60.82^7 cm. as the distance from the film side of the plate to the axis of rotation. The quotient of the distance from the center to the spectral line divided by the distance from crystal to film gives the tangent of twice the glancing angle of reflection and denoting this angle by θ we have

$$\text{Tan } 2\theta = \frac{30.00^0}{60.82^7}.$$

Whence $\theta = 13^0\ 7'\ 35''$, from which by the use of the formula $n\lambda = 2d \sin \theta$, in which n is unity and d has the value 2.814×10^{-8} cm. We find λ to be $1.278^1 \times 10^{-8}$ cm.

TABLE I.

Summary of Results for The L Radiations Wave-Lengths $\times 10^{-8}$ *Cm.*

Line.	Plates 104 and 105.	Plates 115 and 117.	Plate 121.	Plate 122.	Plate 123.	Average.
$\lambda_{1.48}$.....				1.4820	1.4836	1.482^8
$\lambda_{1.47}$.....			1.4719	1.4725	1.4723	1.472^2
$\lambda_{1.41}$.....					1.4163	1.416^3
$\lambda_{1.29}$.....	1.2979		1.2968	1.2976	1.2983	1.297^7
$\lambda_{1.28}$.....					1.2868	1.286^8
$\lambda_{1.27}$.....	1.2781	1.2781	1.2780	1.2784	1.2793	1.278^4
$\lambda_{1.25}$.....	1.2589	1.2580	1.2588	1.2593	1.2598	1.258^6
$\lambda_{1.24}$.....	1.2418	1.2412	1.2413	1.2414	1.2421	1.241^6
$\lambda_{1.22}$.....				1.2205	1.2199	1.220^2
$\lambda_{1.20}$.....				1.2102	1.2094	1.209^8
$\lambda_{1.17}$.....					1.1773	1.177^3
$\lambda_{1.12}$.....				1.1297	1.1286	1.129^2
$\lambda_{1.09}$.....	1.0948	1.0951	1.0948	1.0955	1.0963	1.095^3
$\lambda_{1.07}$.....					1.0705	1.070^5
$\lambda_{1.06}$.....	1.0645	1.0649	1.0643	1.0645	1.0656	1.064^8
$\lambda_{1.05}$.....	1.0587	1.0586	1.0581	1.0587	1.0593	1.058^7
$\lambda_{1.04}$.....					1.0427	1.042^7
$\lambda_{1.02}$.....	1.0250	1.0246	1.0258	1.0250	1.0262	1.025^3
$^1\lambda_{.91}$.....	.9153	.9153	.9158	.9165	.9171	$.915^9$
$\lambda_{.70}$.....				.7058	.7079	$.706^8$
$^2\lambda_{.48}$.....	.4835	.4838	.4838	.4828	.4838	$.483^3$

[1] Wave-lengths shorter than λ .91 are selectively absorbed by the bromine in the plate causing a dark band at the position of this wave-length.

[2] The silver of the plate selectively absorbs wave-lengths shorter than λ .48 thus causing a dark band at the position of this wave-length.

In a similar way the angles of reflection and the wave-lengths were determined for the other characteristic L rays and the results of five separate tests are recorded in Table I. These results were computed from an average of eight separate measurements of each plate. The agreement between the different tests is a fair test of the accuracy of the work since the distances to be measured were different in each case. Table II. gives a summary of the results of different investigators each of whom had either used rock salt crystals directly or had compared the gtating constant of some other crystal with that of rock salt so that in every case the results are based on the value of 2.814×10^{-8} cm. for the

Moseley, Phil. Mag., Vol. 27.	de Broglie,[1] Comptes Rendus, Jan., 1914.	Barnes, Phil. Mag., Sept., 1915.	Gorton, Phys. Rev., Feb., 1916.	Hull, Proc. Nat'l. Acad., May, 1916.	Compton, Phys. Rev., June, 1916.	Siegbahn and Friman, Phil. Mag., July, 1916.	Ledoux-Lebard and Dauvillier, Comptes Rendus, April 30, 1917.	Dershem.
			a_1, Faint, 1.477	a', 1.480	l, 1.5044	α_2, 1.481	9 1.504, Very weak	$\lambda_{1.48}$ 1.482[8], Weak
1.486	Strong ray, 1.471	1.477, Strong	a, Strong, 1.465	a, 1.468	k, 1.4993	α_1, 1.471	8 1.492, Intense	$\lambda_{1.47}$ 1.472[2], Strong
								$\lambda_{1.41}$ 1.416[3], Very faint
					i, 1.3543, Line uncertain			
		1.312, Weak	b_1, Faint, 1.290	b', 1.300	i, 1.3160	β_4, 1.296	7 1.309, Very intense	$\lambda_{1.29}$ 1.297[7], Medium
			b_2, Very faint, 1.283					$\lambda_{1.28}$ 1.286[8], Very faint
	Intense doublet							
	Second ray, 1.279	1.296, Strong	b, Strong, 1.277	b, 1.280	h, 1.2962, Strong	β_1, 1.278	6 1.290, Very intense	$\lambda_{1.27}$ 1.278[4], Strong
	Intermediate, 1.258	1.277, Weak	c, Faint, 1.255	c, 1.260	g, 1.2771	β_3, 1.258	5 1.269, Very intense	$\lambda_{1.25}$ 1.258[6], Medium
	First ray, 1.241	1.258, Strong	d, Strong, 1.238	d, 1.242	f, 1.2587, Strong	β_2, 1.241	4 1.250, Very intense	$\lambda_{1.24}$ 1.241[6], Strong
					e, 1.2349			$\lambda_{1.22}$ 1.220[2], Very faint
								$\lambda_{1.20}$ 1.209[8], Very faint
								$\lambda_{1.17}$ 1.177[3], Very faint
								$\lambda_{1.12}$ 1.129[2], Weak
	Strong ray, 1.096	1.113, Strong	g, Strong, 1.095	g, 1.100	d, 1.1107, Strong	γ_1, 1.095	3 1.101, Intense	$\lambda_{1.09}$ 1.095[3], Strong
								$\lambda_{1.07}$ 1.070[5], Faint
		1.083, Weak	h, Medium, 1.057	h { 1.073 / 1.065	c_2, 1.0796 / c_1, 1.0725 } Line very close double	γ_2, 1.064 / γ_1, 1.058	2 1.067, / 1 1.062 } Weak doublet	$\lambda_{1.06}$ 1.064[8], Medium
								$\lambda_{1.05}$ 1.058[7], Medium
			k, Medium, 1.024	k, 1.0387	b, 1.0539, Line somewhat doubtful			$\lambda_{1.04}$ 1.042[7], Very faint
								$\lambda_{1.02}$ 1.025[3], Medium
	Second band weak, .9192				a, 1.0387			$\lambda_{.91}$ .915[9], Bromine absorption line
	First band (strong, ... .4823							$\lambda_{.70}$ .706[8], Medium
								$\lambda_{.48}$ .483[2], Silver absorption line

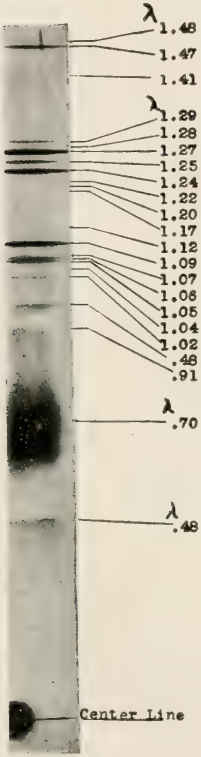

Fig. 5.

Showing the position of the 19 lines of the *L* group and also the boundaries of regions of greater blackening of the plate corresponding to wave-lengths of .9159 and .4833 Ångström units which are due to selective absorption by the bromine and silver of the plate of waves just shorter than their own *K* radiations. (de Broglie, Comptes Rendus, Vol. 158, p. 1493, and Vol. 163, p. 87; Wagner, Annalen der Physik, Vol. 46, p. 868.)

Lines λ .48 are *Ag* absorption lines, the upper one being first order, the lower one second order. Line λ .91 is Br absorption line.

Fig. 6.

ELMER DERSHEM.

distance between the atomic planes in halite. Fig. 5 is a photograph showing the position of the L lines of the tungsten spectrum.

Before doing the preceding work it was thought possible that the distance between planes of atoms in a crystal might not be identical for all crystals of the same substance but might vary with the conditions of growth of the crystal. To test this some preliminary measurements were made using crystals of halite obtained from different parts of the earth. The results showed that to within the limits of error of measurement there was no variation of the grating constant.

Experimental Results for the K Radiations.

In securing the photographs of the K radiations the same methods were followed as in the case of the L radiations except that a higher potential was required. On account of the great penetrability of these rays the use of a thin crystal was much more imperative. Fig. 6 shows a photograph of the four K lines of tungsten. Owing to the use of a thin crystal these lines are all clearly separated in the first order. Other observers using a thick crystal have found difficulty in separating the two lines of shortest wave-length in the first order. Table III. gives the results of four tests for the wave-lengths of the K lines of tungsten and

Table III.

The K Radiations of Tungsten.

Wave-Lengths in Ångström Units.

	Plate 58.	Plate 109.	Plate 114.	Plate 119.	Weighted Average.
$\lambda_{.21}$	.2121	.2126	.2118	.2126	$.212^4$
$\lambda_{.20}$	.2075	.2075	.2069	.2078	$.207^6$
$\lambda_{.18}$	.1833	.1818	.1831	.1837	$.183^4$
$\lambda_{.17}$	.1784	.1786	.1778	.1785	$.178^4$

Table IV.

A Comparison of the Results obtained by Different Investigators of the K Radiations of Tungsten.

Wave-Lengths in Ångström Units.

de Broglie, Comptes Rendus, April, 1916.	Hull, G. E. Review, July, 1916.	Ledoux-Lebard and Dauvillier, Comptes Rendus, December, 1916.	Dershem.	
α_K .2032	α .212	α_1 .2128	$\lambda_{.21}$	$.212^4$
	α .208	α_2 .2053	$\lambda_{.20}$	$.207^6$
β_K .1768	β .185	β_1 .1826	$\lambda_{.18}$	$.183^4$
		β_2 .1768	$\lambda_{.17}$	$.178^4$

Table IV. gives a comparison with the results of other observers. In Table III. in finding the weighted average the last plate is assigned a weight of three and the others a weight of unity since they were not so perfect as the last. In these tests the distance from crystal to plate varied slightly for the different plates, but was always between 60 and 61 centimeters.

Accuracy of the Measurements.

Since the extreme variation from the mean value is not greater than 0.1 per cent. for any characteristic line of the L group the probable error is less than this amount. In the same way the probable error for the K lines is less than 0.8 per cent. On account of the smaller angles these cannot be so easily measured as the L lines.

There is very little possibility that the lines observed may in part be due to impurities in the tungsten target. I have no direct information in regard to the purity of the latter but understand that no impurities can be shown by chemical analysis.

These results agree well with such results as are reported by Siegbahn and Friman and also with those computed from the values of the reflection angles as given by de Broglie but disagree with most of the others. This is to be expected in some cases. Gorton used a film wrapped onto a cylindrical surface. It would seem possible that the film might either shrink or stretch in the process of development. Compton recorded the deflections of an electrometer photographically on a moving film. This gives a graphical representation of the relative intensities of the different lines but it would be difficult to get a precise measurement of wave-length in this way since the angular position of the crystal is not accurately known at the moment when the electrometer deflection is being recorded by the photographic film.

Theoretical Considerations.

Considerable work has already been done, notably the work of Moseley,[1] in correlating the X-ray spectra of the different elements but little progress has been made toward determining whether, or not, the lines of a single element might be grouped into series such as some of the spectral lines in ordinary light are grouped to form the well-known Balmer's series. The theoretical work of Bohr[2] shows that these series in the case of some of the lighter elements may be derived from a theory of atomic structure and it is the belief of many that X-rays are to the

[1] Phil. Mag., Vol. 26, pp. 1024–34, and Vol. 27, p. 703.

[2] Phil. Mag., Vol. 26, pp. 1–25, pp. 476–505, and pp. 857–75.

heavier elements what light rays are to those of lesser atomic weight. If X-rays are produced by the change of motion of electrons near the central nucleus it might be possible to work back from an empirically derived series to the mechanism by which these rays are excited. So far such a series has not been found, but this may easily be due to the fact that so far only a comparatively small number of lines has been found. The failure to find them is more probably due to a lack of resolving power rather than to the existence of but few lines. In the case of the plate giving 19 lines in the L group the resolving power was less than 170 and we know that with such low resolving powers we would have learned but little of that which we now know of light spectra.

By the use of Bohr's theory Kossel[1] has attempted to explain the origin of the K and L radiations by assuming several stable orbits of different radii near the nucleus and that the hardest of the K lines is due to the falling of an electron from the outer to the inner orbit. These theories led to the conclusion that the difference in frequency of the two K lines (at the time he wrote the K lines were treated as only two but these are now known to be double lines) should be the frequency of the L line of longest wave-length. This has been said to hold true for a number of elements, but if we take the average wave-length of the K doublets as found in this work we should have

$$\frac{1}{.1809} - \frac{1}{.2100} = \frac{1}{\lambda L}.$$

Whence $\lambda L = 1.30$ instead of 1.48 Ångström units as it should if the theory were correct. This is a greater variation than is permissible, even granting the greatest possible errors in these measurements.

Summary.

1. This work shows that accurate wave-length measurements and the separation of close doublets can only be achieved by limiting the thickness of the crystal and the width of source and making the distance between crystal and photographic plate as great as is practicable with regard to the necessary intensity.

2. The L group of the tungsten X-ray lines by these means is shown to contain at least 19 lines and measurements correct to 0.1 per cent. are given of their wave-lengths. From considerations of the resolving power of the apparatus it seems possible that the true number may be as great as the number of lines in the light spectra of an element.

3. It is shown that the K lines of tungsten may be clearly separated

[1] Ber. d. Physik. Gesel., Vol. 12, p. 953, 1914.

in the first order if the conditions required for the highest practicable resolving power are complied with.

In conclusion I wish to thank the staff of the physics department and especially Professor G. W. Stewart, who directed the work, for many helpful suggestions and encouragement in the carrying out of this task and also to Mr. A. M. McMahon, who gave much assistance in the performance of the work.

Physics Laboratory,
University of Iowa,
December, 1917.

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THE EFFECTS OF GASES AND METALLIC VAPORS ON THE ELECTRICAL PROPERTIES EXHIBITED BY SELENIUM CRYSTALS OF THE HEXAGONAL SYSTEM.

BY W. E. TISDALE.

THE photo-electric effect of certain crystals and metals has been observed to vary with the freshness of their surfaces, and with the different kinds of gas surrounding them.[1] It has also been suggested that perhaps the well-known action of light on selenium might also vary with the surface conditions, even though it is generally accepted that the change in selenium due to light is a volume, and not a surface effect.[2]

The following experiments were undertaken to ascertain what effect, if any, various vapors or gases would have on crystals of selenium. It was necessary to make crystals all under exactly similar conditions as to pressure, temperature, and rate of cooling upon removal from the incubator, in order to establish their uniformity. A set of eight tubes of selenium was evacuated at one time; sealed off separately, and placed in a Frease oven regulated for 185° C. Before sealing off the tubes, and while they were being evacuated, the selenium was boiled for about fifteen minutes to drive off all the gases in the stick selenium. The crystals were formed in sixty-nine days.

Of these eight tubes, three were picked at random for test crystals. The others were filled with the different gases and put aside that the crystals might soak in the various gases and come to a saturization point. The gases used were hydrogen, argon, nitrogen, oxygen and air. Air was taken as a standard for comparison. The vapors used were mercury, tin, bismuth, sulphur and arsenic.

THE EFFECT OF GAS ON SELENIUM CRYSTALS.

Before attempting to ascertain whether the gas in which the different crystals had soaked produced any change in their electrical properties, *i. e.*, their resistance, sensitiveness to light, maximum wave-length sensibility, rate of action under light, and rate of decrease of resistance upon removal of light stimulus, it was necessary to determine whether crystals

[1] T. Wulf, Ann. der Physik, 9, pp. 946–963, 1902. Kunz and Stebbins, PHYS. REV., Vol. 7, p. 62, 1916.

[2] F. C. Brown and L. P. Seig, Phil. Mag., Vol. 28, p. 497, 1914.

made under similar conditions acted alike when subjected to similar treatment after production.

For this point, crystals from various parts of each of the three test-

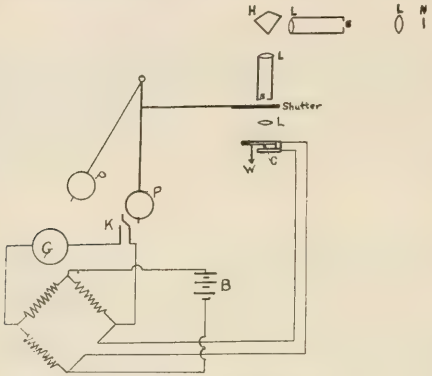

Diagram of Apparatus.

tubes were selected, and their properties determined. The crystals chosen were taken as near the same size as was possible to select them without microscopic measurements. The mean specific resistance for the test crystals was found to be 8.5×10^5 ohms per sq. mm. In Fig. 1

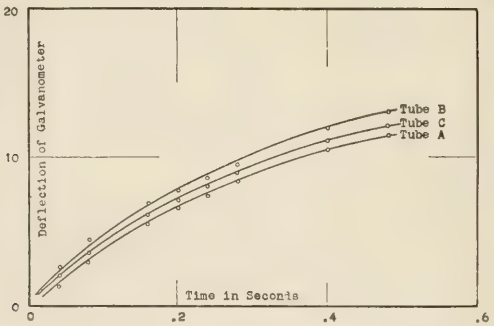

Fig. 1.

Action curve of test crystals.

is shown the action curves for these crystals. They were obtained by placing the crystals in the contacts shown at *C* in the diagram of apparatus, where the crystal was in the dark, and then subjected to the action of light for intervals varying from .04 second to .48 second, and measuring the resultant change in resistance. The change from equilibrium in the dark to that in the light was then plotted as a function of the time. It is seen that in general the slope of the curves are the same, indicating that the rate of decrease of resistance with time under given light condi-

tions is constant. For the recovery to normal dark conditions after being exposed to the light, a constant α was calculated from the theory of recovery proposed by Dr. F. C. Brown.[1]

This theory assumes a certain similarity between the theory for recombination of negative and positive electrons in gases and the recombinations of negative electrons and positive residues in selenium crystals. Rutherford[2] has shown that if in a gas n_1 negative and n_2 positive ions are present, the rate of recombination is directly proportional to $n_1 \times n_2$ expressed by the equation:

$$\frac{dN}{dt} = -\alpha(N_1 \times N_2),$$

where α is a constant called the coefficient of recombination. It assumes that for short intervals of time after the light has been cut off that the decrease in conductivity of a selenium crystal is due to the recombination of the conducting electrons with the positive residues, and that this recombination takes place according to the same law as that which holds for gases. If the number of electrons taking part in the conduction is proportional to the conductivity, the rate of recombination may be calculated from the equation:

$$\alpha = -\frac{dC}{C^2 dt},$$

where dC is the decrease in the conductivity during the time dt that the crystals have been in the dark; C is the equilibrium value of the conductivity in the light; and α is proportional to the coefficient of recombination. It is therefore necessary to assume:

1. When a crystal is in equilibrium in the light or in the dark the conductivity is proportional to the number of negative electrons taking part in the conduction.

2. Recombination takes place in selenium crystals according to the same law as in gases.

3. The rate of recombination in the light is the same as in the dark.

The constant α was measured for crystals from the tubes of test crystals, and were found to be for

tube A . . . . . 2.68×10^5,

tube B . . . . . 2.90×10^5,

tube C . . . . . 2.84×10^5.

These values are in very good agreement.

[1] PHYS. REV., Series 2, Vol. 5, p. 395, 1915.

[2] Phil. Mag., Ser. 5, vol. 44, p. 422, 1897, and Ser. 5, vol. 47, p. 109, 1899.

In Fig. 2 is shown a further similar relation between these crystals from the tubes of test crystals. These curves are a relation between resistance and wave-length of incident light, and show what is known as wave-length sensibility relations.

They were obtained by placing the various crystals in the same position *C* in the apparatus diagram and subjecting them to the same spectrum,

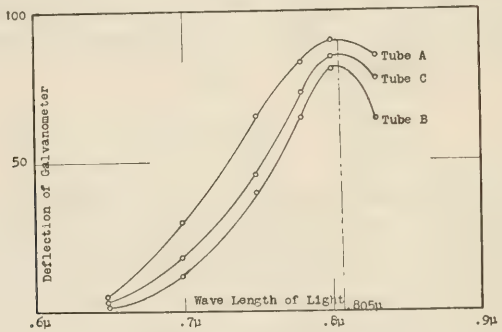

Fig. 2.

Wave-length sensibility curves of test crystals.

produced by a Hilger monochromatic illuminator, where this spectrum was not an equal energy spectrum for all wave-lengths, but was the same energy for all crystals at any given wave-length. The curves are of the same shape and show uniformly a maximum wave-length sensibility at a little further in the red end of the spectrum than .8μ—roughly about .805 μ.

From the various tests here applied, it is justifiable to assume that the crystals in the entire set of tubes made at the same time under identical conditions of pressure, temperature, and rate of cooling, would show identical properties of resistance, light action, recovery from light, and wave-length sensibility if they were all treated alike after production. And further, that if any difference be detected after subjecting them to different treatment, that this treatment is the cause of the difference found. These crystals will therefore be assumed to be normal crystals made in vacuum and tested in air, and the various other crystals will be compared to them. The results will not be referred to absolute measurements, but will be entirely comparative. Only one crystal from each tube will be used in the comparisons, but each one has been checked against many others selected at random from the same tube to be certain that it is not an isolated crystal.

In Fig. 3 is shown the action curves for the different crystals. They were obtained as were the corresponding curves for the test crystals,

but show a marked variation from the similarity noticed there. It will be observed that the crystals treated with nitrogen show a decidedly greater slope than any of the others, and the fact of a more rapid rate of change of resistance with light stimulus, as these curves indicate, is very

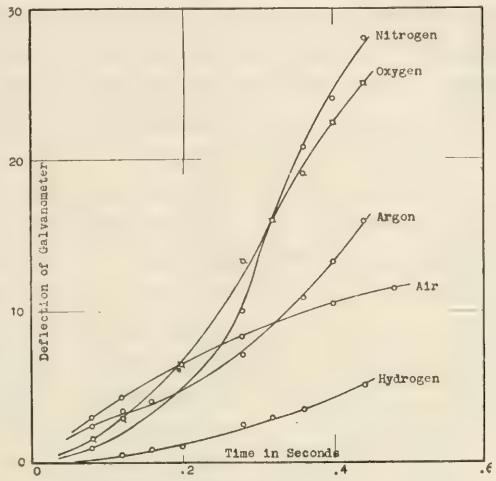

Fig. 3.

Action curves.

noticeable in working with the crystals. The hydrogen-treated crystals are the most sluggish over long periods of exposure, as well as for the short periods indicated in the curves. The air crystal is almost identical with those used as a standard.

Fig. 4 gives the rate of recovery data from which the coefficients of

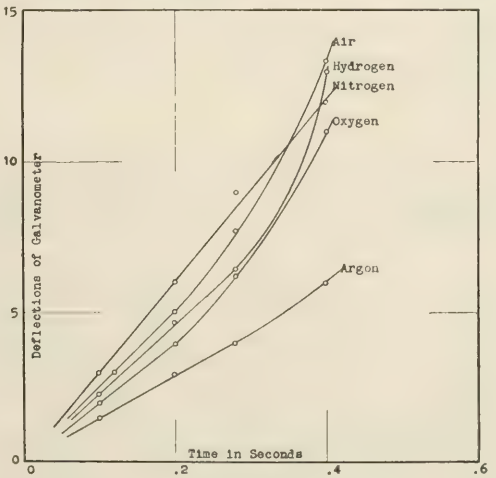

Fig. 4.

Recovery Curves.

recovery were computed. For short intervals of time the nitrogen-treated crystals again show a more marked rate of change than do the others. Argon crystals are very slow in their recovery after exposure to light.

The wave-length sensibility curves in Fig. 5 are particularly interesting. It is to be noticed that nitrogen crystals have changed at their maximum about fifty per cent. more than the comparison crystals changed. In white light, the nitrogen crystals changed their resistance so that the ratio of resistance in the light to that in the dark was 3.4, while the ratio for the comparison crystals was 3.2. Oxygen ratio was 2.8, hydrogen 3, and argon 2.5. These curves show that in monochromatic light the ratio of resistance change is of the same order—nitrogen showing the largest ratio, and the argon crystals the smallest.

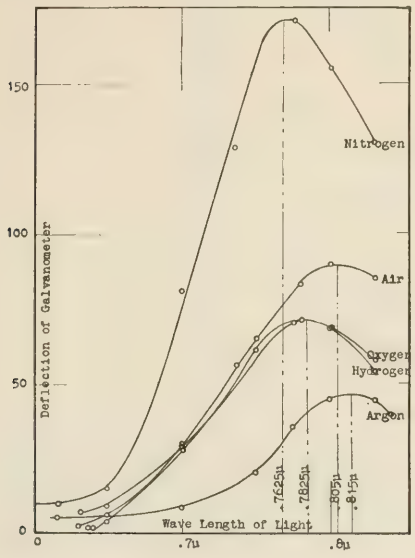

Fig. 5.

Wave-length sensibility curves.

These curves also show a decided difference in the wave-length at which the various crystals reach a maximum. The observations show decided maxima at the wave-lengths indicated. There seems to be no reason why these different maxima should show the law of progressive variation with the gases that the resistance and other effects show.

TABLE I.

	Wave-length Sensibility. (From Fig. 5.)	α.	$R_d \div R_L$.	Specific Resistance.	Atomic Weight.
N.....	1	10.3×10^5	3.4	1.16×10^5	14
Air.....	2	2.68×10^5	3.2	8.5×10^5	
O.....	3	1.7×10^5	2.8	7.5×10^5	16
H.....	4	1.37×10^5	3	14.5×10^5	1
Ar.....	5	1.07×10^5	2.5	9.2×10^5	40

In Table I. is recapitulated the various facts observed. In the first column the crystals are arranged according to their amount of change in resistance at their maximum wave-length sensibility, as shown in Fig.

5. In the second column, values of α are shown, and in the third, the factor showing the ratio of their resistance in the light to that in the dark is given. The fourth column shows the specific resistance of the various crystals, but is subject to considerable error due to the smallness of the area of contact. It is to be noticed that the crystals showing the greatest change in resistance from light to dark also show the greatest coefficient of recombination, as indicated by α. They also show the greatest change at their most sensitive wave-length, and in general, the least specific resistance.

It seems that of these gases, nitrogen improves the action of the crystals far more than do any of the other gases, and that argon does the most damage. Now from the work of Kunz and Stebbins[1] and others, who have shown that argon is one of the most effective gases in photo-electric effects, it seems correct to conclude that there is no relation between photo-electric effects and light sensitiveness in selenium, as is shown also by F. C. Brown in some recent unpublished results.

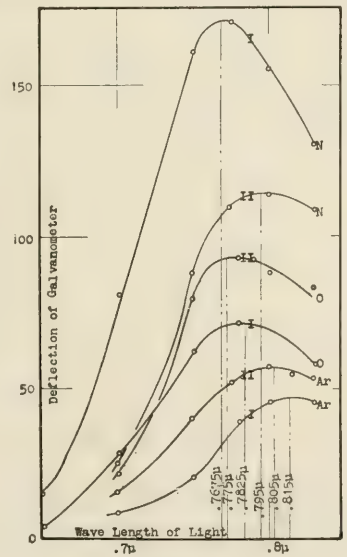

Fig. 6.

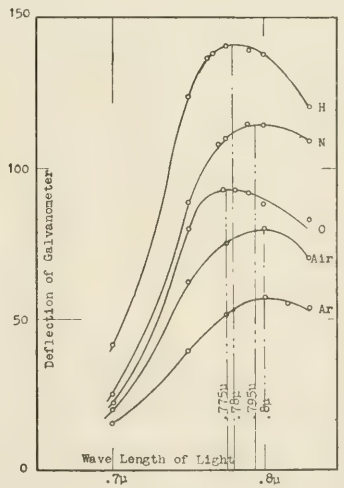

Fig. 7.

Wave-length sensibility curves of crystals taken from vacuum to gas to air.

Inasmuch as nitrogen appears to be the most helpful of these gases, the other gases seem also to be arranged in order naturally, because air, containing four fifths nitrogen, about one fifth oxygen, and small parts of hydrogen and argon, should be next to nitrogen, and should be better than the others, as is shown to be the case. Also, oxygen should come

[1] Phys. Rev., p. 62, Vol. VII., 1916.

third, as it does. Hydrogen and argon follow in their order. Now arranged according to atomic weights, except for hydrogen, these gases also occur in order—the heavier gas being more detrimental than the lighter ones. Hydrogen may be peculiar in its action here as it is in its action on platinum.

After the various tests before given were made, the crystals were placed in separate glass containers, and allowed to stand in dry air ninety days, when they were again tested for maximum wave-length sensibility, and for the values of α. In Figs. 6 and 7 these last curves are compared with those shown in Fig. 5. The curves of Fig. 5 are marked *I* and those of the last tests are marked *II*, and are presented to show the different degrees of change in the differently treated crystals. It will be noticed that the crystal taken from the vacuum tube to air (Fig. 7), has changed but very little, either in sensitiveness or wave-length at which a maximum occurs. The hydrogen crystal has changed more than the others—its change in conductivity is about 100 per cent., and its maximum now occurs at .7825 μ instead of .78 as formerly. The conductivity of the nitrogen crystal has been reduced by a factor of two thirds, and its maximum shifted from .7675 μ out to .795 μ. The oxygen and argon crystals have changed, as can be observed, but in less degree. The principal point for these two is the shift of the maximum (Fig. 6). The values of α shown in Table II. are of interest. These show, as do the curves in Fig. 8, a tendency towards the value of the crystals observed in air throughout the experiment. The recovery of argon has increased by nearly five times, and oxygen two times. Nitrogen has decreased to one third its former value, and air and hydrogen have remained nearly as before. In Fig. 8 the curves marked *II* in Figs. 6 and 7 are compared with each other to show the contrast with the same crystals shown in Fig. 5.

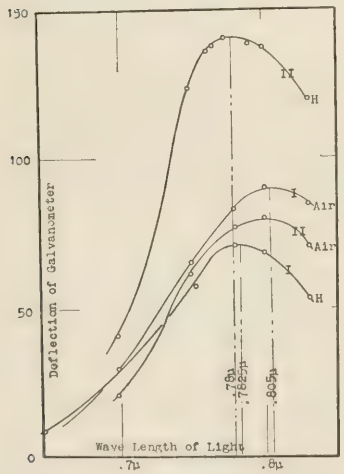

Fig. 8.
Wave-length sensibility curves.

TABLE II.

	After Soaking in Gas.	Ninety Days Later in Air.
N.....	10.3×10^5	3.05×10^5
Air.....	2.68×10^5	2.62×10^5
O.....	1.7×10^5	2.64×10^5
H.....	1.37×10^5	1.65×10^5
Ar.....	1.07×10^5	5.18×10^5

Crystals Made in Nitrogen and Hydrogen.

At the same time that these previous crystals were made in vacuum tubes, selenium was crystallized in an atmosphere of nitrogen, and also of hydrogen. The results were different than for those made in vacuum and soaked in these gases, the reason being that the gas in the tube was at a pressure of nearly two atmospheres, which altered the temperature gradient as well as modifying the pressure under which the crystals formed. The maximum wave-length sensibility for crystals made in hydrogen was the same as for crystals made in vacuum and soaked in hydrogen—both occurring at approximately wave-length .785 μ. There were no remarkable properties of these crystals except that they were very slow of action, as were also those made in vacuum and soaked in hydrogen, requiring at least ten minutes to reach an approximate equilibrium value from dark to light, and requiring hours to recover approximate equilibrium from light to dark.

The crystals made in nitrogen furnished more interesting results. At times they would respond as do the normal crystals, that is, those that in response to light decrease their resistance by certain ratios, depending upon the time of exposure and the intensity of the light. At other times these same crystals would suddenly decrease their resistance while remaining in the dark, and upon exposure to light in this condition, would increase their resistance. There seemed to be no good explanation of these effects, but it had been previously observed that during the process of baking the crystals there had developed in this tube a small crack. The probability of effect by gases was dismissed, but the possibility of the entrance of metallic vapor from the heating coils of the oven suggested a series of experiments to determine their effect on selenium crystals.

The Effect of Metallic Vapors on Selenium Crystals.

A tube of selenium was prepared as previously described for making crystals, except a small tube filled with filings of pure tin was put in the larger tube along with the selenium. There was no contact between the selenium and the tin except in the vapor state, both being at a temperature above the melting point of tin. Crystals formed without any visible difference in color, size or general appearance. Their resistance was of the same order of magnitude as for those prepared in the usual manner—about 2 to 3 $\times$ 10^5 ohms per millimeter cube. However, they were strikingly different from normal in that they were absolutely neutral to the action of light within the limits of the galvanometer which had a sensitiveness of 10^{-9} amperes. A 500-watt nitrogen-filled lamp at a

distance of 50 centimeters had no effect on the conductivity of any of these crystals.

When sulphur replaced the tin in the small inner tube, the crystals were exceedingly irregular, and very short and thick. In general they were decidedly grayish in color. Their resistance was several times as much as normal crystals, and their resistance change with light was of the same order as these vacuum-made crystals. Their maximum sensibility occurred at .83 μ, with very sluggish recovery and action rates. The remarkable fact about the experiment with sulphur was the number of red crystals produced. These red crystals were exceedingly small—too small for definite analysis—but when viewed through a microscope their angles could be definitely seen. When viewed through crossed Nichols they appeared a brilliant red, indicating a double refracting power—a character purely crystalline. When tested for conductivity, to the limits of the apparatus, they showed none. Red crystals of selenium produced by sublimation are particularly rare, having been, so far as known, obtained but once, and those by F. C. Brown.[1]

When bismuth replaced tin, there was no apparent deviation from the normal crystals.

Normal crystals made in vacuum and tested in air were placed in metallic vapors, and their resistance changes observed. The crystals were arranged so that they were not heated as the vapors were drawn over them by means of a pump. All the metallic vapors showed the same effects. If the vapor was drawn over them for too long a period, a thick coating deposited, and there was no other change in the crystal's apparent resistance than a decided lowering, due entirely to the conducting film of metal, and there was no light sensitiveness. If, however, a sufficiently thin layer were deposited by varying the time of exposure to the vapor, for small currents, the resistance would be very low—as for example a crystal whose dark resistance was 3.9×10^6 ohms, when placed in tin vapor for four hours, showed a resistance of 9.9×10^3 ohms. In its original state the crystal was light positive, but in the second state it was light negative. Its negative action was not steady, however. This result was typical of the action of the vapors of bismuth, arsenic and mercury.[2]

There is a possible explanation of this action. It is well known[3] that if a small V-shaped trough has a fine thread of mercury in the bottom, and a current be sent along it, there is a tendency for the mercury to form globules, which breaks the current. This tendency depends upon

[1] PHYS. REV., N. S., Vol. IV., No. 2, Aug., 1914, p. 85.

[2] F. C. Brown, PHYS. REV., N. S., Vol. II., No. 2, p. 153, Aug., 1918.

[3] E. F. Northrup, Metallurgical and Chem. Eng., Jan., 1913.

the size of the current and the size of the thread of mercury. Now with a very thin layer of the metal on the crystal, a small current which is shared by both the crystal and the thin layer will not cause the layer to ball up, but upon subjecting the combination to light, there is enough additional current permitted to flow by the action of the selenium to cause the layer to ball up and break the circuit through it, thus causing all the current to go through the crystal. This, in turn, due to the high resistance of the crystal, causes an apparent light-negative action, *i. e.*, an increase of resistance due to light, instead of the normal decrease. Such a continuous action would cause such fluctuations of the currents as are observed. If one of these crystals showing varying resistance features due to a layer of condensed metallic vapor be connected in the apparatus shown in the diagram of apparatus, and an electro-magnetic wave be sent through the room by the spark from an induction coil, the resistance will instantly increase to approximately its normal value, and for a particular one tested, it required ninety-six hours to again reach a low resistance, having remained undisturbed meanwhile. The action of the electro-magnetic wave is to send a very appreciable current through the layer (the circuit acting as an antenna), which breaks up the film to a larger extent than can occur under the action of light.

Summary.

1. It has been shown that selenium crystals of the hexagonal system show identical properties when they have been subjected to the same conditions during and subsequent to their production.

2. That their various light electrical properties may be altered by subjecting them to the action of different gases. Whether these gases, in a gas layer, change crystal stresses, or vary the energy received from a constant source is not determined.

3. That the effect of the gas seems to be more detrimental the higher the atomic weight of the gas.

4. That the effect of making crystals in gases is but little different than making crystals in vacuum, and subjecting them to gas.

5. That the effect of metallic vapors is to form a conducting film on the surface that may, under certain conditions, give an apparent light-negative action to crystals that are normally light positive.

Physical Laboratory,
University of Iowa.

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THE EMISSION AND ABSORPTION OF PHOTOELECTRONS BY PLATINUM AND SILVER.[1]

BY OTTO STUHLMAN, JR.

AS the result of light passing through very thin metal foil, photoelectrons will be emitted from both sides of the metal simultaneously. Since it will be necessary to distinguish between those electrons which are liberated from the side upon which the light is incident and those that come from the side from which the light emerges, we shall refer to them as "incidence" and "emergence" rays respectively.

It was hoped that some relations could be established between the atomic properties of metals and their photoelectric emissivity. Secondly to establish the magnitude of the coefficient of absorption of the electrons as a function of their speed and atomic properties of the metal in which the absorption was to take place. Finally to use the above information, if adequate, to throw some light on the structure of the atom.

All previous work by the writer and other investigators[2] who have been interested in the photoelectric properties of very thin metallic foil, or still thinner and transparent films of metal was undertaken with cathode deposited films supported by quartz.

The photoelectric currents and velocity investigations were, however, found to depend too much upon the previous history of the cathode and the potential gradient under which the cathodic deposit had been made to warrant any quantitative conclusions.

Thus in many cases, as in those investigations by the writer and K. T. Compton, both the magnitude of the current and the velocity of emission of the electrons could be altered at will, depending upon the impurities that are present through occlusion, in the cathode. The most serious difficulty, however, was found in mounting the very thin transparent films in such a way as to insure perfect electrical contact. These sources of error were so great that no quantitative conclusions could be drawn from the data at that time.

[1] Presented at the Amer. Phys. Soc. meeting, December 27, 1917.

[2] Stuhlman, Phil. Mag., 20, p. 331, 1910; 22, p. 854, 1911; PHYS. REV., 4, p. 195, 1914. Stuhlman and Compton, PHYS. REV., 2, Sept., 1913. Kleeman, Proc. Roy. Soc., 84, p. 92. 1910. Robinson, Phil. Mag., 23, p. 542, 1912; p. 115, 1913; V., 32, p. 421, 1916. Werner, Ark. Mat. Ast. Fys., 8, No. 27, p. 1, 1913. Partzsh and Hallwachs, Annalen XLI., p. 247, 1913. Compton and Ross, PHYS. REV., 9, p. 559, 1917.

In the present work it was found necessary to investigate metallic films of less than 10^{-6} cm. in thickness with a degree of accuracy much beyond that heretofore attained in the above experiments. It was necessary to provide films of extreme purity and make electrical contact with them which would exclude the errors introduced through direct contact clamping between these very thin films and their supports leading to the electrometer. Heretofore successive thicknesses of films were produced either through successive additions of metal to an initial thin film, or successive plates were covered with various thicknesses and then separately investigated. This necessitated the investigation of many films of varying thickness deposited on the assumption that a linear relation existed between time of sputtering, and thickness of metal deposited. To attain this experimental linear relation by keeping the working conditions constant enough in the cathode-depositing-chamber through control of vacuum and cathode fall is itself quite a difficult undertaking.

Transparent Metallic Wedges were Deposited.

In order to obviate the necessity of examining a large number of transparent films of successive increasing thickness, or of examining one film which had been built up of successive layers which had been photoelectrically fatigued through long use, the feasibility of substituting a transparent or semi-transparent metallic wedge was considered.

The deposition of such a wedge by an electrolytic method was however rejected. The difficulties of this method of deposition may be appreciated since we learn from Kundt's[1] well-known experiments on the optical constants of metals, that out of several thousand attempts only a score of wedges were produced which were sufficiently perfect to warrant investigation. A further objection however arises. A chemically deposited metal would contain impurities through occlusion of the electrolyte with the subsequent formation of polarized electrical double layers, which may become so prominent as to prevent all electronic emission.

A metallic wedge deposited by evaporation in vacuo from a fine wire, heated to luminescence, was eventually found to meet the above requirements and objections.

The method[2] essentially consists of a wire, heated to incandescence by means of an electric current, while the quartz plate upon which the wedge is to be deposited is placed below and to one side of it. If the whole is now placed in vacuo, the metal will vaporize and condense upon the quartz surface and surrounding objects.

[1] See Kayser Handbuch der Spectroscopie, IV. Band. p. 542.

[2] Stuhlman. Presented before the Amer. Phys. Soc., New York meeting, December, 1916. Also Jour. Optical Soc. of Amer., V., 1, p. 78, 1917.

Under similar conditions Langmuir[1] has shown that the rate of evaporation m of an incandescent body is expressed by the relation

$$m = \sqrt{\frac{M}{2\pi RT}} \cdot p,$$

where M is the molecular weight of the vapor and p its vapor pressure. He has also shown that no reflection of molecules of the metal takes place at the surface upon which they initially strike, unless the surface is so hot as to cause reëvaporation. Under the above conditions we can therefore conclude that the total amount condensed per second is equal to the loss per second which the wire undergoes through evaporation.

Hence given a wire whose cross-sectional area is small compared with its distance above the plane, over which it is placed horizontally, it may be treated as a linear source which is sending out metallic vapor in all directions at right angles to its surface. Since no reflection takes place from the horizontal plane under the working conditions, then $dm/2\pi$ can represent the amount of evaporated metal passing through unit angle per second and subsequently condensed from a length dl of wire. Let the wire be stretched parallel to and at a height h cm. above the horizontal plane in which the quartz plate is placed, upon which the deposition is to form. After n seconds let t_0 represent the thickness of the metal condensed on the plane just below the wire. The flux per second of metal through an angle $d\theta$ condensing just below the wire producing this thickness t_0 is then

$$t_0 h d\theta = \frac{dm}{2\pi} d\theta.$$

The simultaneous condensation at any other point in the plane at a perpendicular distance r from the wire is given by

$$t_n ds = \frac{dm}{2\pi} d\theta.$$

Since

$$t_0 h d\theta = t_n ds = \frac{t_n r d\theta}{\cos\theta}$$

it follows that the thickness of deposit at any point in the plane is

$$t_n = \frac{t_0 h^2}{h^2 + x^2}, \qquad (1)$$

where x represents the distance of the point n from the foot of the perpendicular h let fall from the wire to the plane. If we set $h = 2a$ and $t_n = y$ the equation reduces to the well-known curve

[1] Phys. Rev., V., 2, p. 340, 1913.

$$y = \frac{8a^3}{x^2 + 4a^2}, \qquad (2)$$

the "Witch of Agnesi," so that a section through the solid deposit formed from an evaporating wire, at constant temperature, with vacuum at constant pressure can be represented by the above curve. The deposit just below the wire is however not a wedge. It has a pronounced curvature concave downward, followed by a curvature concave upward. Further along, that is for the values of x ten times larger than values of y, the curve approaches a straight line. It was in this region that the quartz plate was placed, which was to receive the metal, where the deposit would approach nearest to the desired form of a true wedge.

Apparatus used to Deposit Wedges.

Fig. 1 shows a diagrammatic representation of the apparatus. Although originally[1] designed to make interferometer or opaque mirrors of uniform texture and optical purity, it was found to answer the requirements for the present purpose. The working parts are mounted on a thick plate of rolled brass, grooved to hold the bell jar *BJ*. Hard sealing wax floated in this groove formed the seal for the vacuum which was maintained by means of a Gaede rotary mercury pump. The carriage *HE* supports the incandescent wire. At *H* the wire is attached to a screw at the top of the rod connecting the two wheels, from where it passes down and then through a large hole in the rod between the wheels, then across through a similar hole in the rod at *E*, thence to the spring brass hook, *E*. This spring hook supplies the necessary tension to the wire. Upon heating the wire, it expands and the slack is further taken up by the spring in the hook. For the heating current a 110-volt alternating current was used. The quartz plate was placed in the plane *P* below and to one side of the wire.

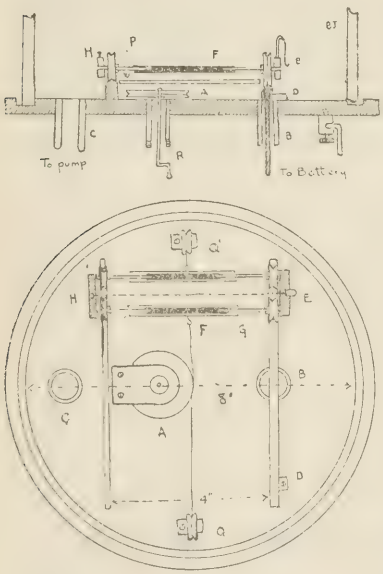

Fig. 1.

[1] Read as a paper before the Amer. Phys. Soc., New York, December, 1916. Jour. Optical Soc., V., 1, p. 78, 1917.

The heating-current enters the base plate at the binding post to the right. It then passes into the metallic track at H which is screwed to the base plate. From here it passes through the left side of the carriage, thence through the wire across to E into the right-hand track. This track is insulated from the base-plate by means of two fiber L-shaped supports. The current passes from this insulated track through the rod B back to the battery. The rod B passes through a hole in the base-plate and is sealed in place with hard red sealing wax, which serves to insulate it from the plate and at the same time makes the opening air tight.

To prevent the carriage supports from short circuiting the glowing wire, they are separated by hard fiber rods. This joint F was then covered with a tight-fitting glass tube extending one centimeter beyond each end of the fiber insulation. This prevented the fiber from being coated with a metallic deposit which would otherwise short circuit the glowing wire. In order to shift the carriage it was necessary to have a mechanical control outside of the apparatus. (A magnetic control would be simpler for the present purpose.) It was supplied by means of a small crank which passed through a ground glass stopper sealed in the tube at R. This crank turned the horizontal V-grooved wheel A, around which passed a belt to the small wheel Q, from where it passed to a hook on the carriage at F. The other end of the belt passed over a similar wheel at Q' and thence to a similar hook on the other side of the carriage. By turning the crank R, the carriage and wire could be rolled to any position above the plane P which contained the quartz plate upon which the metallic wedge is deposited.

In order to insure a very pure deposit it was found necessary to acid clean the wire thoroughly. It was then mounted and heated in vacuo, but not so high as to produce evaporation. This removed most of the occluded gases and such impurities which would otherwise have been condensed on the quartz plate. The wire is now set at the proper distance from the quartz plate and allowed to vaporize until the desired thickness of metal has been attained.

A metallic deposit condensed as outlined above, was first examined for change in thickness as the distance from the wire increased; with a view of ascertaining how near photometric measurements could determine changes in thickness. A glass microscope slide 10×1 cm. was placed directly beneath and 1 cm. below the wire and the metal was deposited until the region just below the wire was opaque. The deposit was next examined by means of a microphotometer with unresolved light from a tungsten filament lamp. A beam 1×10 mm. was directed normally on

the metallic side and after passing through the wedge was focussed on a linear thermopile in vacuo. The throw of a ballistic galvanometer in series was used to measure the current, which was assumed to indicate, in a rough way, the change in thickness of the wedge as successive millimeter portions were passed before the slit of the thermopile.[1] No quantitative conclusions were expected from these data due to the change in the optical constants with change in thickness of the film.

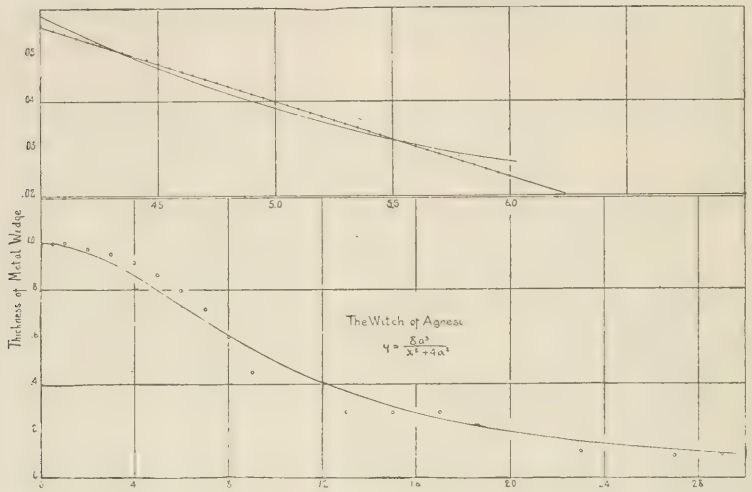

Fig. 2.

These more or less approximate results are shown in the lower curve of Fig. 2. The agreement between the observations as recorded and the theoretical equation $y = 8a^3/(x^2 + 4a^2)$ plotted, as a full line, are as close as might be expected under the circumstances.

In order to determine how near to the wire the best approach to a true wedge could be expected, the following graphical method was resorted to. The theoretical curve was plotted on large scale coördinate paper and that part of the curve chosen which fitted nearest to a straight line within an average of five per cent. The upper graph in Fig. 2 shows the portion of the curve chosen. The solid line represents values computed from the theoretical equation, for the thick end of the wedge at a distance of four centimeters from the foot of the normal let fall from the wire to the plane. The broken graph is a straight line drawn through the average value of the points. Although the metal deposited at this distance from the wire has a slightly concave surface, yet it approaches a straight

[1] I am indebted to Dr. Ives, of the United Gas Improvement Co., Philadelphia, for the use of this apparatus, and to Dr. Eckhardt for assistance with these observations.

line wedge with not more than a maximum of five per cent. variation at any point. This five per cent. error, contributed by the straight line assumption, is within the error of observation contributed by the subsequent photoelectric current measurements.

The metallic wedges investigated in this paper were all deposited in this region.

The Wedges.

The metals investigated were platinum and silver. They were deposited from chemically pure wires .043 cm. in diameter. The wire was 12 cm. above the plane which contained a $2.5 \times 1.0 \times 0.1$ cm. quartz plate placed four centimeters away from, with its narrow side parallel to, the incandescent wire.

The platinum wedges, when examined by reflected light, showed for the opaque deposit at the edge nearest to the glowing wire, the characteristic metallic lustre of the metal. Semi-opaque thicknesses transmitted red light and reflected blue. As the thickness decreased the reflected light changed from blue to brown, to yellow, to blue and at the very thinnest end to indigo. The colors of the transmitted light for the same thicknesses were gray brown, greenish blue and reddish brown.

Silver wedges which were opaque at one end and shaded to zero thickness at the other end in 4 cm. of length showed the following variation in color. Deep violet was transmitted through the nearly opaque thicker end, shading into violet, blue, light blue, deep blue, purple, red, brown and into yellow at its thinnest end. The corresponding points when viewed by reflected light showed at the opaque end the metallic luster of silver followed by deep green, green, green blue, red green, red, blue red, blue and a lighter blue at its thinnest end.

An absorption band might hence be expected to exist in the green part of the spectrum about λ 4500. An absorption spectrum photograph obtained by means of a Hilger monochromatic illuminator through one of the silver wedges, showed a continuous gradually increasing absorption through the red and yellow, terminating at about λ 5300, from here a weak broad green blue absorption band exists terminating at about λ 4000 from where the absorption again drops off regularly into the violet but at a greater rate than the ascent in the red end.

While the thick and nearly opaque ends of some of the platinum wedges are transparent to red light (in fact the glowing filament of a tungsten lamp was visible through the thick end as a brilliant red image); the thick or nearly opaque end of the silver wedge showed the same filament as an indigo-blue image. Showing that the shorter wave-lengths penetrate further into a silver surface than into platinum. What evidence the

photoelectric observations throw on this topic will be discussed in a separate chapter.

The Arrangement of the Apparatus.

The metallic wedge (1 × 2.5 cm.) supported by the quartz plate is shown in Fig. 3 as *Q*. It was mounted with the thick end of the wedge attached to the brass holder *E*. A countersunk screw pressed the metal-covered side of the quartz plate rigidly against the opposite facet of the clamp at *E*. This clamp was screwed to a rod ending in a rack *B*, which slid under V-shaped grooves in the fixture *A*. The pinion *N* engaging the rack, pushed the wedge into any desired position with respect to the beam of light *T* which was used to excite the photoelectric phenomena. The position with respect to the illumination was indicated on the steel millimeter scale *S* fastened to the fixture at *A*. This motion of the wedge allowed one to expose successively increasing or decreasing thickness of the metal to the beam of ultra-violet monochromatic illumination passing through a Hilger Monochromatic Illuminator placed in front of the slit *R*. A 110-volt Cooper Hewitt quartz mercury arc running under constant power supplied the illumination. The slit at *R* was so adjusted that a clear cut image 8 mm. long and 0.23 mm. wide fell on the quartz plate at *T*. The degree of sharpness of the image and its position was at all times controlled by means of a fluorescent plate which could be substituted for *Q* and by means of a fluoroscope eye piece placed at *O*. The photoelectric chamber *D* is an oxidized copper cylinder covered with camphor soot whose photoelectric constants were well known, having been repeatedly used in former experiments. This cylinder D was mounted coaxially with the outer brass casing *P*. It was connected as shown, to an electrometer after passing through the hard sealing wax support at *P*. The electrometer showed a consistent sensibility of 1,220 divisions per volt which did not vary more than 1.3 per cent. during the course of the experiments.

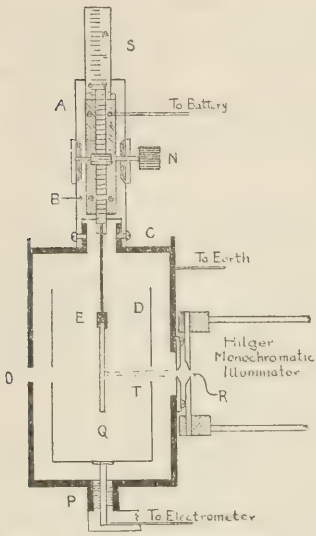

Fig. 3.

The present method differs fundamentally from previous measurements on thin metal films in so far as photoelectric current can be mea-

sured from one film, possessing various thicknesses and having the same history of production and ageing. An additional improvement consists in being able to examine the very thinnest film without introducing special appliances or precautions for electrical contact. It also allows one to measure the thickness of the very thin films to a greater degree of accuracy through extrapolation from the thicker films, which in turn lend themselves more easily to accurate measurements.

The wedge was exposed at millimeter intervals through its entire length. The present paper deals with the photoelectric current obtained from such a wedge when the metal side faced away from the light. Under these conditions the illumination was allowed to fall under normal incidence on the quartz side of the plate, it then passed through the transition layer quartz-metal and liberated photoelectrons in the metal. The electrons leaving the metal under these conditions are classified as "emergence." To measure the "incidence" effect all that was necessary was to rotate the superstructure in the groove *C* through 180° and repeat the measurements for thicknesses as relatively determined by the scale *S*.

Film Thickness.

The determination of the thickness of the film contributed the largest error to the final results. Optical methods as developed by Wiener[1] were given up as too unreliable. A very promising method, though not used on the data presented below, and especially adaptable to silver films, is that in which the silver film is locally converted into silver iodide by means of the proximity of a minute piece of iodine. A convex lens placed upon the resulting hump allows one to use the Newtonian ring interference method for computing the thickness of the hump and from it a value of the thickness of the original silver.

Here either mechanical or electrical methods were resorted to. As in the case of platinum, when the specific resistance of the material is known in terms of thickness,[2] it was found feasible to determine the thickness at several points along the wedge by using the following indirect method.

A quartz plate, similar in size to the one upon which a wedge was deposited, was covered with a uniform film, of thickness comparable to the thick end of the wedges. This uniform film can be deposited in the apparatus discussed above by simply moving the carriage, supporting the incandescent wire, back and forth across the plate with uniform speed. After attaining its equilibrium value the film was examined for its spe-

[1] Wied. Annalen, V., 31, p. 629, 1887.

[2] Patterson, Phil. Mag., IV., p. 663, 1902. Longden, Phys. Rev., II., p. 40, 1900. I. Stone, Phys. Rev., VI., p. 1, 1898.

cific resistance. This resistance data could be used to compute its thickness. A photoelectric examination of this uniform surface made under similar conditions as existed when the wedge was investigated, determined the thickness of that part of the wedge which showed an equal photoelectric current.

When the thickness of the plane film was large enough, the above method was checked by direct weighing. The plane silver films could be changed to the iodide form, and then weighed, and from these results the thickness can be computed.[1]

EMERGENCE PHOTOELECTRIC CURRENT OBSERVATIONS.

The photoelectric currents for wave-lengths 2260, 2301, 2536 and 3131 were obtained for platinum and silver with variations in thickness of the metal exposed. Since the various spectral lines are not of equal

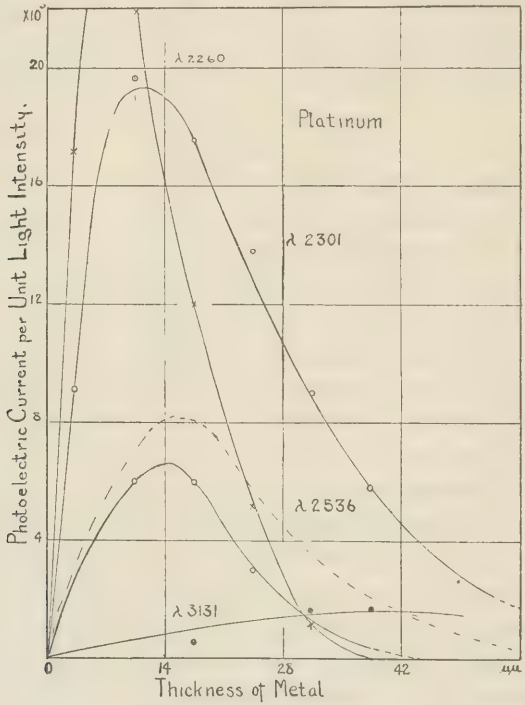

Fig. 4.

intensity the observations were reduced to represent current per unit incident light intensity, by dividing the number of electrons liberated per unit time by the energy of the light of the corresponding wave-length.

[1]Quinicke, Pogg. Ann., 129, p. 178, 186 .

The ordinates thus plotted in subsequent curves represent the relative number of electrons liberated in unit time by a unit amount of light-energy for the wave-length indicated.

In the case of platinum an attempt was made to get some results from λ 3131 but with little success, since platinum is no longer photoelectric sensitive for this wave-length. These latter results are only introduced for qualitative comparison, and since no light filter was used, no doubt the observed current was not really due to the excitation from this wave length, but from a somewhat shorter wave-length.

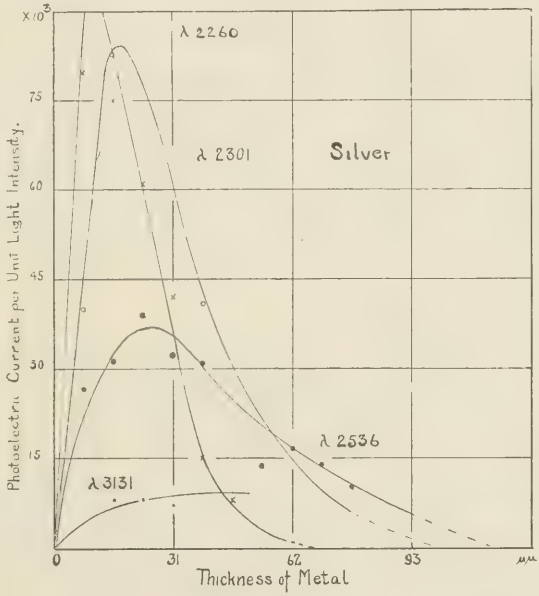

Fig. 5.

Figs. 4 and 5 show a typical set of observations for each of the metals. To avoid the error that might be introduced by photoelectric fatigue it was found necessary to prepare a separate wedge of metal for the observations obtained for each wave-length examined.

The photoelectric observations could be duplicated with a mean error of five per cent., but since the thickness determinations possessed an average error of ten per cent., no additional precautions were introduced into the photoelectric measurements to insure a greater accuracy. The constants computed from these curves show a degree of consistency in agreement with the above errors, which serve as a good check on the observational values.

In Fig. 4 the dotted line for λ 2536 gives the unfatigued values of the

photoelectric current computed from the solid fatigued curve just below it. The ordinates of this curve were obtained by increasing the observed fatigued values by an amount experimentally determinable and proportional to the time elapsing between the removal of the wedge from its vacuum chamber and its subsequent photoelectric examination.

The curves for both metals have this in common; all make finite angles with the abscissa. There is no sign of inflexion on the ascent of the curve, so that the tangent to the curve at the origin is more inclined to the abscissa than the tangent at any other point. This means that for the very thinnest part of the wedge (atomic thickness) the photoelectric effect occurring there for any frequency of the exciting light, bears a definite ratio to the mass per square centimeter of the film. The slopes of the curves at the origin are relative measures of the energy absorbed by a film thickness comparable to the diameter of one molecule.

The curves can, for the sake of discussion, be divided into two parts. The initial rise which appears to be a type of saturation curve, and superimposed upon this a second phenomena of a more complex nature approximately represented by an exponential drop followed by a linear relation ending in another less rapid fall.

These two phenomena will be treated separately under the subject of scattering and absorption of photoelectrons.

The photoelectric current is excited by a monochromatic beam of light falling normally on the metal surface, and if no electronic absorption takes place the photoelectric current should be proportional to the light energy absorbed. If the optical constants of the metal do not change with change in film thickness for the wave-lengths used, it would follow that as the thickness of the metal increases linearly the photoelectric current should increase exponentially. This would continue until a thickness is attained beyond which the monochromatic light could no longer penetrate. This does not however hold true. When we measure the emergence effect, the curves seem to indicate that the light gives rise to electrons in amount apparently greater than the amount of light absorbed. It becomes therefore necessary to determine the cause of this additional electronic activity.

THEORY.

For the sake of simplicity let us consider only those electrons which leave the atom in the emergence direction, possessing energy proportional to the frequence of the exciting light but independent of the kind of atom from which they initially escape.

As the liberated electrons pass through successive layers of matter,

let us further assume for simplicity that they are absorbed according to an exponential law.

If then a beam of light of frequency ν strikes a metal film, and if I_0 is the intensity penetrating the surface normally, if the intensity has fallen to I after passing through a thickness dx, then

$$I = I_0 e^{-\epsilon x}.$$

If all of the absorbed energy is used in liberating photoelectrons in such a way that the energy absorbed is equal to the photoelectric energy liberated then

$$I_0 - I = I_0(1 - e^{-\epsilon x}), \tag{3}$$

where ϵ is a constant, denoting the rate of appearance of electrons. It depends on the frequency of the light used and may be identified with the coefficient of absorption of the light. Since electronic density and light intensity are proportional under these circumstances, I_0 can be considered a variable, subsequently to be absorbed exponentially in the distance $t - x$. Then

$$\frac{dI_0}{dx} = -\alpha I_0$$

where α is the coefficient of absorption of the electrons passing through thickness $t - x$. The solution of this relation is given by

$$I_o = I' e^{-\alpha(t-x)}.$$

Since all the electrons due to any frequency ν of the light have the same initial velocity and hence may initially be considered a homogeneous beam coming from a layer of metal between x and $x + dx$, it follows if we assume that only those electrons are considered that move off in the emergence direction, that the intensity of the emergence radiation is proportional to

$$\frac{dN}{dx} = \epsilon I_0 e^{-\epsilon x} = \epsilon I' \int_0^t e^{-\epsilon x} e^{-\alpha(t-x)} dx$$

and

$$N = \frac{\epsilon I'}{\alpha - \epsilon}(e^{-\epsilon t} - e^{-\alpha t}). \tag{4}$$

It is seen that for small thicknesses the photoelectric current is proportional to the thickness or number of atoms per cubic centimeter. With increased thickness the curve passes through a maximum when

$$t_m = \frac{1}{\alpha - \epsilon} \log_e \frac{\alpha}{\epsilon} \tag{5}$$

and apparently decreases exponentially with the intensity $I' e^{-\epsilon t}$.

In order to determine to what extent the above curves check this theory the discussion is for convenience divided into two parts, the initial absorption of light as a function of the thickness and the absorption of the photoelectrons as determined by that part of the curve showing the decrease of the photoelectric effect with its subsequent disappearance as the thickness increases.

Absorption of Light with Variation in Thickness.

If under the experimental conditions outlined above, the photoelectric current is represented for any thickness t, by the relation $I_0(1 - e^{-\epsilon t})$ then this is equivalent to saying that the intensity of the electronic radiation from a unit area of the metal is always proportional to the number

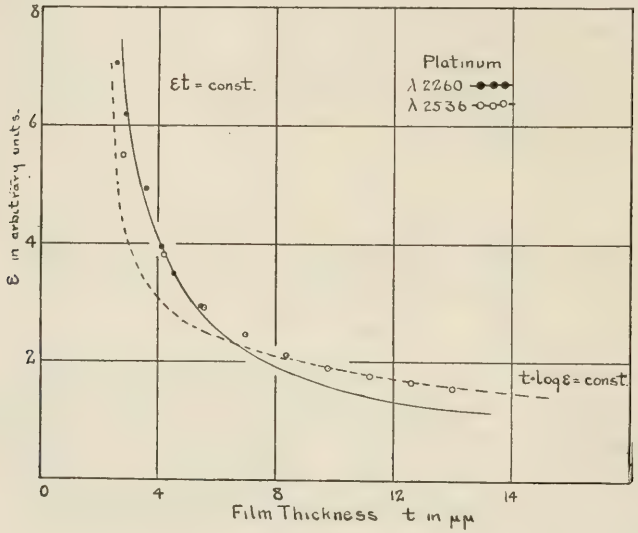

Fig. 6.

of atoms present in one cubic centimeter. The values of ϵ therefore obtained from these curves would help to determine the validity of this relation. If the number of electrons liberated are proportional to the amount of light absorbed, and if the metal film is so thin that the absorption of the electrons as they pass through the successive layers of molecules can be neglected, then successive similar layers of the metal will absorb equal fractions of the amount of light energy they receive, provided no scattering takes place, and emit a quantity of electronic energy which is a measure of the light energy absorbed. Under these conditions we can measure the electronic energy and use it as a measure of the light energy absorbed. If then we let ϵ represent this coefficient as measured

by the electronic results, I_0 the intensity of the energy entering the slab of material, I the intensity leaving, then $I = I_0 e^{-\epsilon t}$ where t is the thickness of the slab. This may be rewritten in the form

$$\epsilon = \frac{2.3}{t} \log \frac{I_0}{I}$$

If now $\log I_0/I$ is plotted against t, the graph should be a straight line (for ϵ constant) and ϵ may be determined from the relation $\epsilon = 2.3 \times$ slope.

The curves thus obtained were however not linear, they showed a continuous decrease in the coefficient ϵ (Fig. 6) as the thickness increased. The coefficients were then computed from the photoelectric curves using the relation $\epsilon = 0.693/D$ where $e^{-0.693} = 0.5$ and D is the thickness which reduces the radiation to half value. Here $1/\epsilon$ represents the distance to which the energy penetrates before its intensity is reduced $1/e$ of its original value.

TABLE I.

Platinum, λ2301.			Silver, λ2301.		
t.	ϵ.	ϵt.	t.	ϵ.	ϵt.
1.40 $\mu\mu$	.990 $\mu\mu^{-1}$	1.39	1.70 $\mu\mu$	.393 $\mu\mu^{-1}$	.668
2.10	.660	1.39	2.54	.260	.660
2.70	.520	1.40	3.30	.206	.679
3.50	.396	1.38	4.25	.160	.680
4.20	.330	1.39	5.00	.135	.665
5.05	.283	1.43	5.77	.114	.657
5.90	.230	1.35	6.90	.100	.690

TABLE II.

Platinum, λ 2536.

t.	ϵ.	ϵt.	$t \log \epsilon$.
2.8 $\mu\mu$	.550 $\mu\mu^{-1}$	.154	264
4.2	.381	.160	244
5.6	.291	.163	260
7.0	.247	.173	275
8.4	.211	.177	273
9.8	.190	.186	273
11.2	.176	.197	276
12.6	.165	.208	273
14.0	.157	.216	271

The above extended values give some idea of the general trend of the results. From the samples cited in Table I. it is seen how very close the product ϵt approaches a constant for small values of thickness. The

constancy of this product was found to hold, in the case of both platinum and silver for λ 2260 and 2301, so long as the values thus obtained remained associated with very small values of t. When the thickness became larger ϵt no longer remained constant but increased with increasing values of t. A characteristic set of observations illustrating this condition is cited in Table II. If however, the relation $t \log \epsilon$ is computed from the data it is found that this relation is constant for large values but not for small values of t. As an example the case of Pt λ 2536 is cited, the change taking place at thickness 7 $\mu\mu$.

One characteristic common to all these results is that ϵt seems constant under two circumstances, either for quite a range in values of t for large values of electronic velocities, excited by means of λ 2260 or 2301, or for a very short range in the values of t (about 7 $\mu\mu$) when slower electronic velocities are used, as for example those excited by λ 2536 or 3131. Although the curve for platinum λ 3131 is shown in Fig. 4, yet the coefficients calculated from this curve were omitted owing to insufficient points on the curve to determine its initial slope. In addition the minimum wave-length sensibility for platinum is just inside the wave-length used so that the above curve must have been obtained through part of the shorter wave-lengths in the spectrum overlapping this region. The value for silver λ 3131 has only been introduced to show the order of magnitude of the value ϵt.

TABLE III.

Mean Values of ϵt = *const. for small values of* t.

λ.	*Pt.*	*Ag.*	Ratio.
2260	1.41	.681	2.07
2301	1.39	.671	2.07
2536	1.61	.983	1.64
3131		1.80	
Atomic weights	195.2	107.9	1.82

A summary of these results for values of t small, representing mean values similar to the extended data given above are shown in Table III. The last column gives the ratio of the values for this relation. It is of the same order of magnitude as the ratio of the atomic weights. We might conclude that the light energy absorbed per atom, is proportional to its atomic weight.

Before going into the possible origin of the above relations it may be of interest to examine the constancy of the relation ϵt further. The values of t were therefore plotted against values of $1/\epsilon$ (Fig. 7). Under these conditions one would expect to get a straight line whose slope would

give a relative measure of the absorption of the light in passing through a molecular layer.

Since the amount of light energy absorbed is proportional to the photoelectric energy liberated it follows that the values of ϵ determined for changes in thickness, are also measurements of the photoelectric characteristics. Under these conditions we may view the electron as liberated from its parent atom, passing through successive layers of molecules being deflected through collision with them but never entering the system of the atom to disappear through absorption.

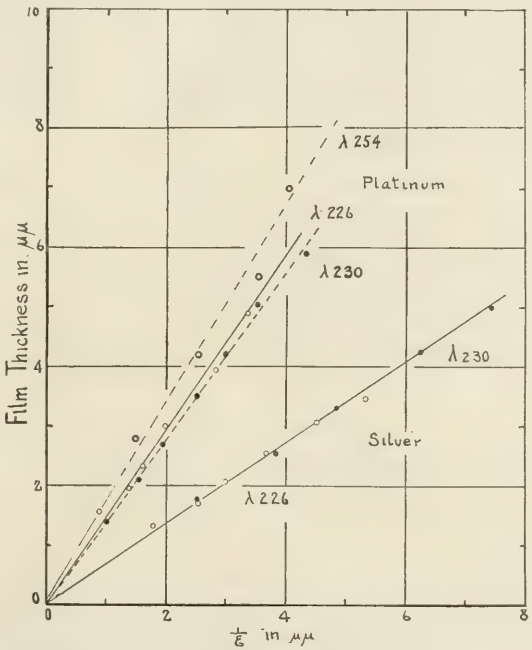

Fig. 7.

If therefore we consider two electrons, liberated with the same initial velocity, in the two metals, we will find after having passed through equal thicknesses of metal that the electron must have undergone a greater energy change per unit distance in passing through platinum than through silver. This ratio is, within experimental error, the same as the ratio of the densities of the two metals. This means that if equal masses are compared the amount of energy scattered is the same in the two cases. A similar relation exists for secondary radiation produced by Röntgen rays.[1] The above results are shown graphically in Fig. 7. In general they also seem to indicate that the greater the velocity of separation of

[1] Barkla, Phil. Mag., 7, p. 543, 1904.

the electron from its parent atom, the less the scattering of its energy as it passes through neighboring atoms. The observations for platinum λ 226 do not fall in line with this conclusion. This exception can however be attributed to experimental error. The results for Ag λ 254 are here omitted so as to prevent too much crowding in the figure. The results for λ 226 and 230 lie so close together that it is impossible to separate them in the graph, though the actual results from which the curve is compiled show that λ 230 lies to the left of the results for the smaller wave-length.

If ϵ represents the fraction of the total light energy lost through absorption and reappearing as electronic energy, in going through a sheet of metal of molecular thickness t, I_0 the intensity of the beam entering the sheet and I the intensity of the beam leaving it, then $1/\epsilon$ the intensity coming out of the sheet is given by

$$\frac{I_0 - I}{I_0} = \frac{1}{\epsilon} = kt, \tag{6}$$

where k is a constant. From which we get

$$I = I_0(1 - kt). \tag{7}$$

For larger values of t under the same circumstances it was found experimentally that

$$t \log \epsilon = \text{const.},$$

then

$$\frac{I_0 - I}{I_0} = \frac{1}{\epsilon} = e^{-c/t}, \tag{8}$$

or

$$I = I_0(1 - e^{-c/t}). \tag{9}$$

These relations do not however conform to our simple theory advanced at the beginning of the paper which involved the assumption of an exponential absorption leading to equation (4).

Theory of Single and Compound Scattering.

Rutherford[1] has advanced a theory of single scattering which leads to the conclusion that the proportion I/I_0 of the rays which remain undeflected after passing through a thickness t, when t is small enough,[2] is given by the relation

$$I = I_0(1 - kt).$$

A relation deduced from the above experimental data for thicknesses less than 7 $\mu\mu$ (Fig. 6).

[1] Rutherford, Phil. Mag., V., 21, p. 684, 1911.

[2] Richardson, Electron Theory, p. 496.

For very small thicknesses we must therefore have electrons passing through the metal film, colliding according to Rutherford's hypothesis of "single scattering" and little or no energy is contributed to the atom through which the electrons passed. From this it is seen that the relation between I/I_0 and t should be nearly linear in the initial stage, and this is found to be the case for all wave-lengths for both metals investigated.

If however the thickness penetrated by the electrons becomes greater a larger number of collisions necessarily results, and then the relation $t \log \epsilon = \text{const}$ expresses the state of affairs. It seems as if the electron might be considered as at first moving in fairly straight lines with little deflection and little or no energy exchange, but as the penetration increases the deflexions become more numerous and violent resulting in the rate of expenditure of energy increasing so rapidly that the electron seems to come to a sudden stop.

This stage of the process seems to be equivalent to what Thomson[1] describes as "compound scattering." He showed that the probability of an average deflexion of an electron on passing through a metal sheet of thickness t, if less than a given angle, is equal to $1 - e^{-c/t}$ where c is a constant for any particular substance, which under the present experimental circumstances means that the proportion I/I_0 of the electrons undeflected is given by $I = I_0(1 - e^{-c/t})$ a relation identical with the empirical equation (9).

The experimental arrangement and method therefore appears to fulfill the requirements for single scattering when the metal films are very thin (less than 7 $\mu\mu$), and compound scattering when the metallic film is thicker (Fig. 6).

In these experiments the angular opening ϕ through which the electrons escape from the film is not varied, and if the same metal be examined so that the absorbing medium is also kept the same and the velocity of the electrons varied through change in wave-length of the illumination both Rutherford's and Thomson's theory leads to the relation,

$$\left\{ \frac{mv^2}{t_0^{1/2}} = \text{const.}, \right. \tag{10}$$

where t_0 is the value of the thickness required to cut down the radiation to half value.

This relation can be tested with film thicknesses in which single scattering takes place. In the experimental arrangement ϕ is kept constant. Now the maximum energy of release of the electron is independent of the kind of atom and only depends on the frequency of the exciting light.

[1] Camb. Phil. Proc., 15, p. 465, 1910. See Richardson, Electron Theory, p. 493.

If therefore we consider values of $1/\epsilon$ for equal values of t_o and λ in the two metals, we have a measure of the relative average energies with which the electrons appear after passing through equal thicknesses. This also gives us a comparison of the distances to which electrons penetrate so as to have their initial energies reduced to $1/e$ of their original value. The results obtained from the data as shown in Fig. 7 indicate that equation (10) is valid within experimental error for film thickness less than 10^{-7} cm.

An interesting and rather important change takes place after the electron penetrates further into the metal. In Fig. 6, we see an illustration of the thickness at which this occurs. We are here confronted with another illustration of the sudden change in the property of a metal when its thickness is less than 10^{-7} cm. Similar changes occur in the optical and electrical properties of these metal films in this immediate vicinity.[1]

Unfortunately not enough data were available from individual results for λ 2536, to show clearly that the smaller values of t followed the ϵt = const. relation (Fig. 6), so that the values for λ 2260 were added to show how close the results fitted the theoretical curves, here shown as solid and broken lines. The latter results were therefore shifted so that the axes of the curve coincided with the axes of the curve for values λ 2536. In this way the results show graphically just where the region of separation exists to which the above theories apply.

If we now consider thicknesses in which compound shattering exists we are limited on one side by the thickness comparable to 7×10^{-7} cm. and the maxima of the curves shown in Figs. 4 and 5. As seen from the sample data shown in Table II. the coefficient decreases less rapidly than the increase in thickness. This is to be expected, if the above theory can be applied; for the amount of scattering becomes more marked with decrease in speed and increase in thickness. This continues until a critical thickness is reached where the absorption of the number of electrons becomes predominant, finally leading to a decrease in the amount of emergence radiation.

The initial rise in the curves can therefore be divided into two distinct regions. For thicknesses less than 10^{-7} cm. the velocities of the emergence electrons must be nearly equal or so nearly that very few if any electrons appear whose energy is less than that demanded by the Einstein relation. Here little if any absorption takes place so that the electron passes through subsequent layers with slight loss of energy. From here on however the number of electrons possessing less than this maximum energy decreases, they undergo frequent and violent collision

[1] Patterson, Phil. Mag., IV., p. 663, 1902. Minor, Ann. d. Phys., 10, p. 581, 1903.

with a rapid loss in energy until some electrons seem to come to a sudden stop. Further penetration is then accompanied by loss in numbers in addition to loss in energy.

ABSORPTION OF PHOTOELECTRONS.

Our theory as outlined at the beginning of the paper indicated a maximum for values of t given by the relation

$$t_m = \frac{1}{\alpha - \epsilon} \log_e \frac{\alpha}{\epsilon}.$$

The values for t_m as picked off of the ionization curves are given in Table IV., Fig. 8. The values of t_m there indicated represent the distances to

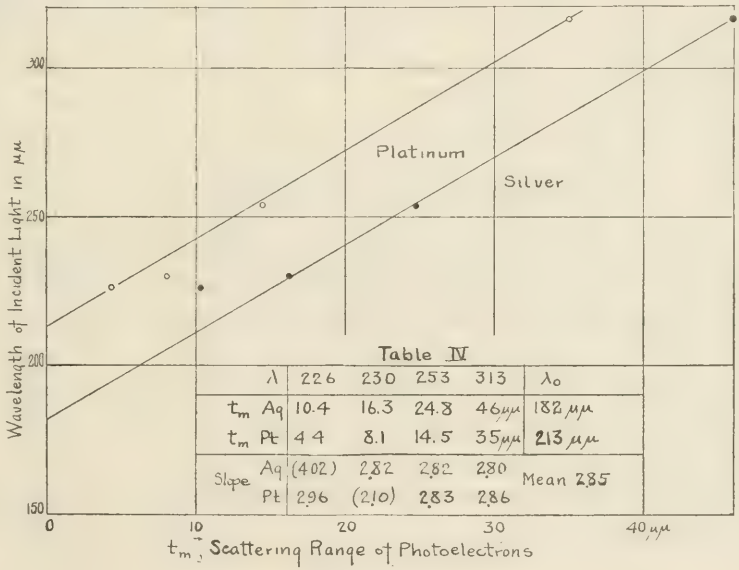

Fig. 8.

which an electron, starting from the lowest layer with initial energy $\frac{1}{2}mv^2 = h\nu$, can penetrate, before it disappears through absorption. The distance t_m is here called the scattering range. It does not measure the actual distance pursued by an electron in the form of the summation of all the zigzag paths, but the distance perpendicular to the surface of the metal. The values obtained in this way are plotted as a function of the wave-length of light causing the separation of the electron from its parent atom. The slope of the curve is the same for the two metals, the bracketed values in the table are however marked exceptions to this rule. The cause for this unusual difference cannot be accounted for through

experimental error. It seems likely however that the straight line relation is perhaps a first approximation to the truth and more numerous data determined for longer and shorter wave-lengths than here investigated will determine to a more exact degree the necessary relations.

The present curves however fit the relation $\lambda - \lambda_0 = at_m$ to a marked degree. Here a is the slope of the curve and equal to 2.85, λ_0 is the value of the minimum wave-length producing electrons possessing an effective

TABLE V.

	Velocity $\times 10^{-7}$ Cm. per Sec.				Mean Velocity.	λ_0.
λ in $\mu\mu$.	226.	230.	253.	313.		
Silver	14.5	14.0	13.3	13.5	13.8×10^7	182 $\mu\mu$
Platinum	14.1	13.8	13.2	13.4(?)	13.6×10^7	213 $\mu\mu$
$v = \sqrt{\frac{2h\nu}{m}}$	14.05	13.95	13.25	11.92	$\times 10^7$ cm/sec	

scattering range equal to zero. This would mean that λ_0 would be the minimum wave-length limit which could produce a photoelectric current under the above experimental conditions. Higher frequences might liberate electrons, but their energies would be so great that they could penetrate the next atoms in their path and completely disappear in their system. These electrons would be absorbed so promptly that we could expect a metal to possess the property of selective absorption for electrons possessing these critical velocities. The minimum wave-lengths of the incident light energy producing electrons under these conditions are 213 and 182 $\mu\mu$ for platinum and silver respectively.

If however the above wave-length relation is reduced to its equivalent energy form we have

$$\frac{1}{v^2} - \frac{1}{v_0^2} = bt_m, \tag{11}$$

where $b = 1.42m/hc$, assuming the relation $\frac{1}{2}mv^2 = h\nu$ to hold in the above case. We define m as the mass of the electron h as Planck's element of action and c as the velocity of light. Under these conditions v would be the velocity the electron must possess, to be able to penetrate a thickness t_m. Table V. shows the initial velocities the electrons should have, to penetrate thickness t_m, as computed from equation (11) and the velocities computed from the Einstein photoelectric equation. The contact difference of potential is here assumed equal to zero, since t_m is defined as the distance the electron travels in the metal without passing through the contact difference of potential layer. This use of the value

of t_m is however not rigidly true but owing to lack of further information it is offered as an approximate solution. We see in Table V. to what extent the values for the computed velocities check the experimental values. In all cases the velocity decreasing with increased wave-length of the exciting energy.

Now $1/t_m$ may be defined as the stopping power. So that the stopping power of the metal increases as the energy of the electron increases, to eventually approach a limiting value. Comparing these values we note that the heavier the atom the greater the stopping power. If we therefore compare the distance traveled in platinum to that in silver for electrons of equal initial velocity we find that the distance traveled in silver is 1.17 as great as that in platinum when the energy in each is reduced to the same amount. Hence if we compare equal volumes of the material traversed we find that the ratio of the atomic volumes of silver to platinum is 1.13 or of the same magnitude as the relative distances traveled in the two metals. So that electrons after traversing equal masses have their initial energies reduced to the same extent.

Absorption Coefficients.

After passing through a thickness t_m the photoelectric thickness curve falls off slowly at first, then nearly linear and finally more slowly again

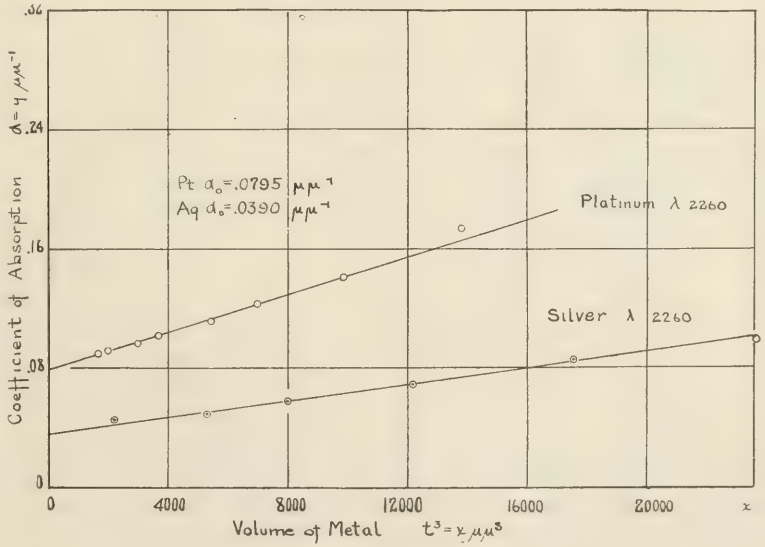

Fig. 9.

cutting the thickness axis at what appears to be a finite value. Unfortunately the latter part of the curve was very difficult to determine

owing to the very minute number of electrons coming through such thick metallic layers.

The initial drop of the curve seems to indicate that there is little absorption of electron numbers as yet and that scattering is still predominant. From here on however the curve takes a very definite form. The problem is a very complex one, the steam of photoelectrons which are traveling in any given direction must suffer loss in numbers both through true absorption and scattering to such a degree as to completely disappear from the original direction and in doing so must lose energy as well. What the relative importance of these actions is in the present case is difficult to tell. A partial solution is however attempted.

The coefficients of absorption α were computed in the usual way for the absorption branch of the curve. These results were then plotted as ordinates and values of t, as abscissæ. The curve approximated nearest to a straight line when values of t^3 were used as shown in Fig. 9. Thus the relation $\alpha - \alpha_0 = mt^3$ would represent the relations to a comparative degree of accuracy limited by the experimental errors in the original observations. The values computed for α and values for m, the slope of the curves, are given for comparison, to show to what degree of accuracy the results may be valid.

TABLE VI.

Platinum, λ 226.			Silver, λ 226.		
α.	t.	m.	α.	t.	m.
.167 $\mu\mu^{-1}$cm^{-1}	24.0 $\mu\mu$	63.0 $\mu\mu^{-3}$cm^{-3}	.099 $\mu\mu^{-1}$cm^{-1}	29.5 $\mu\mu$	23.4 $\mu\mu^{-3}$cm^{-3}
.141	21.5	62.8	.083	26.0	25.0
.123	19.2	62.3	.070	23.0	25.4
.112	17.5	61.6	.059	20.0	25.0
.102	15.5	61.9	.050	17.5	20.5
.0972	14.5	60.0	.043	12.8	20.0
.0917	12.6	61.2			
.0907	11.8	68.0			
	Mean	62.6 $\mu\mu^{-3}$cm^{-3}		Mean	23.2 $\mu\mu^{-3}$cm^{-3}

The coefficients of absorption thus obtained are not constant, but increase very rapidly with increasing values of thickness traversed. The intercepts on the y axis are however of interest. They would represent the absorption coefficients for values of thickness commensurable to a molecular layer. These values of α_0 for platinum and silver were found to be .0795 and .0390 $\mu\mu^{-1}$ respectively for photoelectrons emitted with a velocity of 13×10^7 cm. per sec. from their parent atoms. If we divide

the above values by the density of the respective metals to which they apply we arrive at the following interesting relation

$$\left(\frac{\alpha_0}{\rho}\right)_{Pt} = 37{,}000 \text{ cm}^2\text{gm}^{-1},$$

$$\left(\frac{\alpha_0}{\rho}\right)_{Ag} = 37{,}100 \text{ cm}^2\text{gm}^{-1},$$

showing the mass absorption to be constant for λ 226. This is the "density law" of absorption at first suggested by Lenard.[1] Crowther,[2] experimenting with β rays from uranium showed that the "density law" did not hold for these larger velocities. Lenard experimenting with heterogeneous cathode rays arrived at the following values for the metals gold and silver here given for comparison.

$$\frac{\alpha_0}{\rho} = 2880 \text{ cm}^2\text{gm}^{-1},$$

$$\frac{\alpha_0}{\rho} = 3070 \text{ cm}^2\text{gm}^{-1}.$$

Compton and Ross working with sputtered platinum obtained from photoelectric measurements the average value $\alpha/\rho = 28{,}900$. While Partzsch and Hallwachs (*l. c.*), using some data supplied by Rubens and Ladenburg, obtained the value $\alpha/\rho = 47{,}400$ for gold in the form of a leaf of about 100 $\mu\mu$ thickness.

The exigencies of the times have interrupted the completion of this subject at this time, it is hoped however that in the near future further data can be published which will support the above conclusions. The experimental part of this paper was performed at the University of Pennsylvania and I take great pleasure in acknowledging the ever generous coöperation of Prof. Goodspeed in furnishing the funds for the necessary apparatus.

[1] Lenard, *l. c.*

[2] Crowther, Phil. Mag., 12, p. 379, 1906.

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THE L-SERIES IN THE TUNGSTEN X-RAY SPECTRUM.

BY OSWALD B. OVERN.

SYNOPSIS.

Tungsten X-ray Spectrum, L-series; Wave-lengths.—Dershem's results were verified, for the most part within 1/10 per cent., except in the case of 1.777×10^{-8} cm. which was found to have a wave-length of 1.202×10^{-8} cm. Six new lines were discovered, four of which may belong to tungsten: 1.236, 1.213, 1.079 and 0.793×10^{-8} cm. This last line appeared light in contrast to the dark bromine absorption band in which it was located; an explanation of this fact is suggested. Dershem's method was used with a very thin crystal and with very long exposures of the Coolidge tube. Four photographs are reproduced.

Tungsten X-ray Spectrum, L-series; Two Groups of Lines Suggested.—Sixteen lines fall into two groups, each line of one having a wave-length 1.151 times that of of the corresponding line of the other.

Molybdenum X-ray Spectrum, K Series.—Three faint lines were measured which may belong to this series, wave-length 0.711, 0.706 and 0.629×10^{-8} cm.

Bromine X-ray Absorption Band.—Evidence of discontinuity found.

Silver X-ray Absorption Band.—Evidence of discontinuity found.

Thickness of a Soluble Crystal.—Determined by a simple method.

IN a recent paper[1] Dershem has given the results of photographs of the tungsten X-ray spectrum obtained by the use of a thin crystal of rock salt wherein he finds this spectrum to contain nineteen wave-lengths in the L-series which he claims to have measured to an accuracy of one tenth per cent. The large number of lines discovered for the first time by Dershem made it seem of importance to verify his work. With this in mind, the present experiments were undertaken with the additional object of finding more lines if possible.[2]

The apparatus was the same as that employed by Dershem. The distance of the plate from the axis of the crystal was kept in the neighborhood of 30 cm. Although slits of various widths were used, the best results were obtained with a slit of the same width as that used by Dershem, namely, .032 cm. Except for a few changes in minor details, the method of measuring the plates was also the same as that used by Dershem. Seed's X-ray plates were used.

The departures from Dershem's method were as follows:

1. The length of exposure was made from two to six times as great

[1] PHYSICAL REVIEW, N.S., Vol. XI., June, 1918, p. 461.

[2] For accurate measurements of the *L* series of tungsten by various investigators the reader is referred to citations in Dershem's article, *loc. cit.*

as the longest exposure used by Dershem. The actual length of exposure ranged from 10 hours to 31 hours per degree of rotation of the crystal, the power input being kept the same as in Dershem's work.

2. The thickness of the paper envelope enclosing the plate was reduced to half.

3. A thinner crystal was used, the crystal used for most of the work having an average thickness of .0077 cm. This was measured as follows:

The thickness of the crystal and glass to which it was fastened with wax was measured with a micrometer caliper. The crystal was then dissolved in cold water and the thickness of the glass and sealing wax again measured. The difference was considered the thickness of the crystal.

No difficulty such as that described by Dershem due to the cracking of crystals when ground down to small thicknesses was experienced. Indeed one crystal was ground down to a thickness of less than .003 cm. and was used successfully in making a photograph of the spectrum. This photograph is shown in Fig. 4. The fogging of the plate seems to be materially diminished by the use of a very thin crystal.

Experimental Results.

The wave-lengths corresponding to the lines found on the various plates are given in Table I. Most of the plates were exposed over a limited portion of the spectrum and hence all the lines are not found on any one plate.

Where two plates are listed together they have been exposed on opposite sides of the apparatus and averaged. This, however, does not mean that every line appeared on both plates. The number of plates on which each line was observed is therefore placed in parentheses immediately at the left of each wave-length. The final averages are not weighted but each value is counted once for each plate on which that line appeared from which that value was calculated. Each plate, in turn, represents from five to ten separate observations.

Average values of wave-lengths greater than 1.0×10^{-8} cm. have an estimated accuracy of one tenth per cent. Others are estimated to be accurate within two tenths per cent. Dershem's values are placed in the last column for comparison.

These results not only verify Dershem's work but also show the existence of six more lines not observed by Dershem. If we add to this list the L-series line found by Siegbahn[1] which has a wave-length of about 1.66×10^{-8} cm., the total number of lines exclusive of the K-lines

[1] Deutsch. Phys. Gesell., Verh. 18, 5, pp. 150–153, 1916.

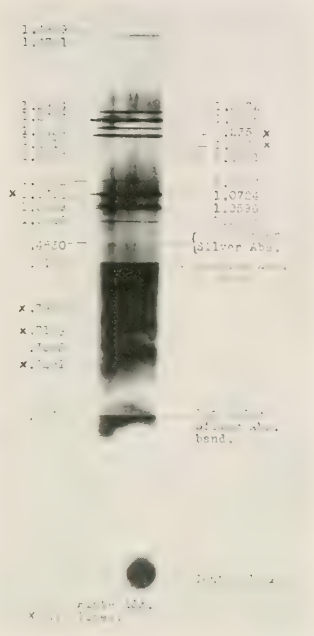
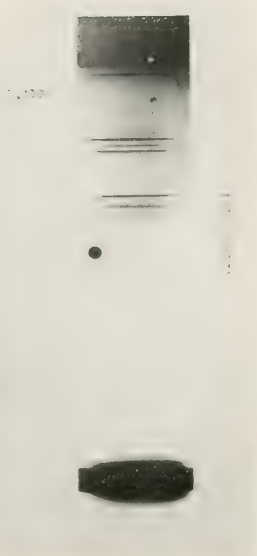

1.0724
1.0369
Silver Abs.
Band.

FIG. 1.

1.2384
1.2213
1.2021
1.0724
1.0369
× New lines.

FIG. 2.

Bromine Absorption
Band
.7708
.7063
Silver
Band
Center



is found to be 25. Whether or not these are all due to tungsten is however open to dispute as will be shown below.

TABLE I.

Wave-lengths in 10^{-8} *cms. Grating constant for rock salt* $= 2.814 \times 10^{-8}$ *cm.*

Plates 202, 203.	Plates 205, 206.	Plates 210, 213.	Plate 214.	Plate 216.	Average.	Dershem's Values.
(1) 1.484^{0}				1.483^{8}	1.483^{9}	1.482^{8}
(2) 1.473^{1}			1.473^{5}	1.472^{6}	1.473^{1}	1.472^{2}
(1) 1.299^{2}	(2) 1.298^{5}			1.296^{5}	1.298^{4}	1.297^{7}
(1) 1.287^{6}	(2) 1.287^{0}			1.287^{1}	1.287^{2}	1.286^{8}
(2) 1.279^{4}	(2) 1.280^{8}		1.278^{3}	1.277^{0}	1.279^{3}	1.278^{4}
(2) 1.260^{3}	(2) 1.260^{6}		1.259^{0}		1.259^{8}	1.258^{6}
(2) 1.242^{5}	(2) 1.245^{4}		1.241^{2}		1.243^{4}	1.241^{6}
(1) 1.235^{3}	(2) 1.234^{8}			1.238^{0}	1.235^{5}	
(1) 1.221^{4}	(2) 1.221^{6}			1.220^{3}	1.221^{2}	1.220^{2}
(1) 1.213^{3}	(2) 1.213^{6}			1.212^{5}	1.213^{2}	
(1) 1.209^{7}	(2) 1.210^{3}			1.208^{4}	1.209^{7}	1.209^{8}
(1) 1.202^{0}	(2) 1.202^{4}			1.201^{8}	1.202^{1}	1.177^{3}
(1) 1.130^{1}	(2) 1.130^{4}			1.129^{9}	1.130^{2}	1.129^{2}
(2) 1.096^{6}	(2) 1.097^{8}		1.094^{9}	1.096^{2}	1.096^{7}	1.095^{3}
(1) 1.079^{2}	(2) 1.079^{5}				1.079^{4}	
(1) 1.071^{9}	(2) 1.072^{6}				1.072^{4}	1.070^{5}
(2) 1.065^{7}	(2) 1.066^{8}		1.064^{6}	1.066^{0}	1.065^{9}	1.064^{8}
(2) 1.059^{7}	(2) 1.060^{4}		1.058^{0}	1.059^{5}	1.059^{6}	1.058^{7}
(1) 1.043^{0}	(1) 1.046^{3}				1.044^{6}	1.042^{7}
(2) 1.025^{9}	(2) 1.027^{0}		Plate 211	1.025^{7}	1.026^{3}	1.025^{3}
		(2) $.792^{8}$			$.792^{8}$	
(1) $.711^{4}$		(2) $.709^{9}$	$.709^{9}$	$.712^{9}$	$.710^{8}$	
(2) $.707^{4}$		(2) $.705^{6}$	$.705^{5}$	$.707^{7}$	$.706^{5}$	$.706^{8}$
		(2) $.629^{1}$			$.629^{1}$	
Br. (2) $.916^{8}$		(2) $.915^{7}$		$.919^{2}$	$.916^{8}$	$.915^{9}$
Ag. (3) $.483^{4}$		(1) $.481^{9}$		$.484^{4}$	$.483^{0}$	$.483^{3}$

The positions of all these lines in the spectrum are shown in Figs. 1, 2, and 3.

It will be noticed that the agreement between the present results and those of Dershem is within one tenth per cent. save in the cases of the wave-lengths, 1.243^{4}, 1.096^{7}, 1.202^{1}, 1.072^{4} and 1.044^{6}. In the case of the first two, this difference can be explained by the fact that these lines, being very intense, are overexposed on my plates with a consequent widening of the image. Since the long wave-length edge of the line is measured, over-exposure will make the results a trifle too large. The discrepancy cannot be explained in this way for the other lines since they did not appear to be overexposed. The line 1.202^{1} is listed by Dershem as 1.177^{3}. Since Dershem's value was found from observations on only one plate and mine from observations on four plates which agree well together (see Table I.) it is probable that Der-

shem's value for this wave-length is in error far beyond his limit of experimental error.

The wave-length 1.416[3] listed by Dershem has not been included in Table I. because it is the writer's opinion that this is the second order of the wave-length .706[5] found on most of the plates. This second order line has appeared on only one of these plates (see Fig. 4) sufficiently strong to admit of measurement. This plate was exposed in a small apparatus and the measurement is not considered as accurate as the others. The value found is 1.419. If this is a second order line its true wave-length is .709 which does not compare so badly with the wave-length .706[5]. The line .706[5] forms quite a close doublet with the line .710[8] and it would therefore be reasonable to expect both lines to appear in the second order if one does. However, the intensity of the line .710[8] is so small in comparison to that of the line .706[5] that it is very difficult to bring it out even in the first order and hence it could not be expected to appear in the second order. A reference to Fig. 4 will show that the observed line is very faint.

The line of wave-length .792[8] appears light on the plates in contrast to the others which appear dark (see Fig. 3). There may be two explanations for this:

1. It may be the edge of an absorption band due to some substance through which the rays pass before reaching the plate. The rays pass through glass, air and paper. The edge of the absorption band would be at the short wave-length end of the spectrum of some element contained in these substances but the K-series wave-lengths of these substances are far too large for this region. The "J radiations" for carbon, oxygen and aluminium, evidence for which has been found by Barkla and White,[1] are far too short for this region. Therefore this assumption is not very probable.

2. It may be the convergence wave-length of the L-series or some other series in the tungsten spectrum. The end of a series would appear as a dark band with a lighter region at the short wave-length end. This sudden change from dark to light is probably what has been observed here. As seen from the photographs, this line is in the region of the bromine absorption band and the general blackening of the plate makes it hard to observe.

The line of wave-length 1.2132 makes a close doublet with the line 1.209[7] already found by Dershem (see Figs. 1 and 2). The close proximity of these two lines which are clearly resolved shows that the resolving power of the apparatus with the crystal used was at least 336 for that wave-length.

[1] Phil. Mag., 34, p. 270, Oct., 1917.

An examination of the bromine and silver absorption bands reveals several sudden but faint changes in the intensity of the blackening of the plate. This may indicate that this region is rich in spectrum lines. An attempt has been made to measure the wave-lengths of some of these but the work has not progressed far enough to make the results of any value.

Of the lines which stand out prominently in this region, the wave-lengths .710[3] and .706[5] are within two per cent. of the wave-lengths of the K-series alpha lines of molybdenum found by other observers.[1] The wave-length .629[1] is within two per cent. of the value given by Blake and Duane[2] for the edge of the absorption band of molybdenum and hence may correspond to the K-series beta line of molybdenum. It is therefore probable that these three faint lines are the K-series lines of molybdenum produced by the molybdenum used in the construction of the Coolidge tube.

Two Groups in L-series.

A scrutiny of the wave-lengths thus far discovered revealed the fact that they may be divided into two groups which are exactly similar. Since the α line of Moseley is the principal line of one group and the β line holds the same relative position in the other group, they have been called the "α group" and "β group" respectively. Their arrangement is shown in Table II.

Table II.

α Group.		β Group.	
λ.	λ/λ_α.	λ.	λ/λ_β.
1.4839	.993	1.2872	.994
1.4731 = λ_α	1.000	1.2793 = λ_β	1.000
1.2984	1.134	1.1302	1.132
1.2598	1.169	1.0967	1.167
1.2434	1.185	1.0794[3]	1.185
1.2355[3]	1.192	1.0724	1.193
		1.0659	1.200
1.2212	1.206	1.0596	1.207
1.2132[3]	1.214		
1.2097	1.217		
1.2021	1.225	1.0446	1.225
		1.0263	1.246

[1] Kaye, "X-Rays," second edition, Longmans, p. 226.
Also Malmer, Phil. Mag., 28, 1914, p. 787.

[2] Physical Review, X., Dec., 1917, p. 700.

[3] Lines found for the first time in this work.

Table II. is unique and gives evidence to show that there are at least two distinct series of lines in the L-series. The similarity of the two groups is shown by the second and fourth columns. There are two lines missing from each group to make it correspond to the other group which gives a possible suggestion as to where to look for new lines.

Any line in the β group may be found from the corresponding line of the α group by dividing its wave-length by 1.151.

A further discussion of these line groups will be left for a later paper.

In conclusion the writer wishes to thank the Staff of the Physics Department of the State University of Iowa for the encouragement and inspiration received from them and especially Professor G. W. Stewart who suggested the problem. He is also indebted to Dr. Elmer Dershem for his initiation into the technique of the experimental work.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA,
July, 1918.

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PROPAGATION OF SOUND IN AN IRREGULAR ATMOSPHERE.

BY G. W. STEWART.

SYNOPSIS.

Under poor atmospheric conditions, lower frequencies in aëroplane engine sounds become relatively enhanced; under good conditions, frequencies of order of 1,000 d.v. are heard at greatest distances. The former is explained by irregularities in the atmosphere and the latter by characteristics of audition.

Intensity of the sound varies much more rapidly than as the inverse square, crude observations giving much more nearly inverse sixth and fourth powers for maximum ranges under fair and good listening conditions, respectively.

LORD RAYLEIGH'S[1] reference to and explanation of the "highly uneven character of the sound" from aëroplanes in flight, leads the writer to make a record of three additional facts.

During experiments in connection with aëroplane detection and location the observers noticed that under what might be termed "poor listening" atmospheric conditions the sound from the aëroplane when at the greatest hearing distance was limited to the lowest frequencies in the emitted complex sound. These frequencies were for these particular aëroplanes approximately 90, 180, 270, etc., and the most prominent component was the one of lowest pitch. The sound from the same aëroplanes heard at the greatest possible distance under excellent night conditions was distinctly different. The lowest frequencies just named were not noticeable and the sharp cracking sounds of the engine explosions, with prominent components, probably of the order of 1,000, were most distinctly in evidence. The difference in the character of the sound in the two cases may be described as the cutting off of the higher frequencies in the former and of the lower frequencies in the latter. That there may be a more rapid decay of intensity of the higher frequencies is readily understood by a consideration of differences in wave-length. The irregularities in the planity of the strata, for example, would be more effective in scattering, by reflection and refraction, the frequencies having the shortest wave-lengths. In general, the irregularities of the atmosphere would affect the shorter wave-lengths the more.

[1] Nature, Vol. 101, 1918, p. 284.

The apparent better transmission for the higher frequencies in the second instance is not to be explained by any influence of the medium but rather by the characteristics of audition. The sense of loudness for the different frequencies is not the same, whether the intensities are measured in mechanical units or in terms of minimum audibility. It is the latter unit of measurement that is of interest in the present case, for the nature of the sound heard at a great distance from the source depends upon audibility.

Professor Sabine[1] has proved that a frequency of 64 and of 1,024 appear of equal loudness when the former is 7×10^4 times and the latter 15×10^6 times its own minimum audible intensity. This means that the observer might be situated near a source emitting these two sounds and consider them of equal loudness, and yet with a homogeneous atmosphere and the decrease of intensity of sound according to the inverse square law, he could find that as he recedes from the source he would cease to hear the lower frequency first because its intensity has passed below the minimum audible limit. Thus the explanation of the above-mentioned change of quality with distance from the aëroplane under good listening conditions is to be explained on the varying relative intensities required for equal loudness and minimum audibility. According to Sabine's values, given sounds of equal loudness at a given distance, the one of frequency approximately 1,024 will "carry" the farthest in an ideal medium if the ear is used as the receiving instrument, and the frequencies 64 and 2,048 on either side of this "maximum" will be heard approximately the same distance.

The other experimental fact worthy of record is the rapidity with which the intensity falls off with distance in the atmosphere. The experiments here to be mentioned are crude, but are sufficiently accurate inasmuch as the exact conditions of the atmosphere could not be recorded. Occasions were selected when the temperature gradient at the ground was practically zero and a comparison was made of the maximum distance from the unaided ear and from the listening device, respectively, that a complex sound could be heard. This experiment was repeated many times and in every case the maximum distance with the instrument was approximately ten times that with the unaided ear. Conditions were such that the selectivity of the instrument was believed negligible. Inasmuch as the experiments were performed on a heavily grassed prairie the decrease of intensity with distance must have been even more rapid than as the inverse square. Similar listening experiments were made using the same device and with aëroplanes in flight. On fair sunny days,

[1] Contributions, Phys. Laboratory, Harvard Univ., No. 8, 1910.

cumulus clouds forming, with aëroplanes at elevation of one thousand yards, the aëroplanes could be heard only twice as far as with the unaided ear. On days when the atmosphere was obviously more irregular, the decay of intensity was much more rapid. Under good night observing conditions with the aëroplane at an elevation of two thousand yards this maximum distance was increased to three times the distance possible with the unaided ear.

It thus appears that even when the atmosphere is favorable to sound transmission, there is sufficient irregularity to cause a surprisingly rapid decay of sound from an elevated source.

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THE FUNCTION OF INTENSITY AND PHASE IN THE BINAURAL LOCATION OF PURE TONES. I.

BY G. W. STEWART.

PART I. INTENSITY.

SYNOPSIS.

Logarithmic Law.—The only published quantitative experiments giving the effect of intensity alone are those of Hovda and the author who found a "logarithmic law," viz., that the effect of intensity only was an apparent angular displacement from the median plane proportional to the logarithm of the ratio of intensities at the ears.

Logarithmic Law Extended; the Value of the "Constant."—The law just cited is found to be correct for the three frequencies 256, 512, and 1,024 d.v., and for a displacement range from 0° to almost 90°. But some individuals (two out of four) do not have a fused tone or phantom source at the frequency last named and indeed, at other frequencies also. The constant, in the logarithmic law, *i.e.*, the value of the displacement when the logarithm of the ratio of intensities is unity, is ascertained for several individuals at the above frequencies. For each individual the constant *decreased* with *increasing* frequency.

The experimental method is the one previously used. The logarithmic law is shown not to be an extension of Weber's law, well known to psychologists, but a new law.

Computed Values of Intensity Ratios for Given Positions of Source.—The actual intensities obtaining at the ears when a source of sound is placed at different positions relative to the ears are considered and the theoretical results are plotted and compared with the experimental results.

Intensity Not an Important Factor.—The wide divergence between experimental and theoretical values shows that an explanation in terms of intensity of the ability to locate a pure tone, 256 to 1024 d.v., is not possible. Intensity cannot be an important factor in localization of pure tones in this range of frequencies.

I. INTRODUCTION.

A SURVEY of the many reports of experiments in the localization of sound that discuss the function of intensity would not prove of sufficient interest here for, in every instance except those noted, the experimenters have not studied intensity effects with the assistance of quantitative measurements. Indeed, for the most part, writers have been content to discuss what appeared to them must be the effect of intensity differences at the ears. Rayleigh[1] was the first to point out that for frequencies of 128 and 256 d. v. the difference in intensity at the ears could not account for the ability to locate the source of sound.

[1] Rayleigh, Phil. Mag., 1907, 13, p. 217.

The first quantitative measurements of the intensity effect with a pure tone were made by the writer, and Hovda[1] who discovered that if relative intensities at the ears of a pure tone, 256 d.v., were varied, but with the phase difference maintained at zero, there resulted a displacement of the apparent source of the fused sound from the median plane, this displacement, θ, to the right, and the intensities at the ears, I_R and I_L, having the following relation,

$$\theta = K \log_e \frac{I_R}{I_L},$$

wherein K was a constant for an individual. This source of fused sound appeared to have a position external to the head in a plane approximately horizontal, usually in front rather than in the rear, and the displacement occurred in a circle of constant radius.

The purpose of the present paper is to present the effect of intensity only in producing a displacement of the apparent source, or the image,[2] from the median plane, to discuss these results in comparison with intensity relations existing when a similar source is displaced in a circular path about the head, to make note of the limitations of the logarithmic law just cited and to derive conclusions as to the importance of intensity as a factor in the location of pure tones of the range of frequency here considered.

II. Apparatus.

The apparatus and method used throughout these as well as the previous experiments of Stewart and Hovda are similar in all essential respects and have received sufficient description in the article[3] already published. For the sake of clearness it should be briefly stated that the source of the sound was a tuning fork, that the observer used stethoscope binaurals and sat at the center of a circular scale in order to ascertain the angular displacement of the image, and that the intensities were varied by altering the position of one of the receiving tubes at the fork without producing a phase difference, and that the relative intensities at the ears I_R and I_L were ascertained by means of a Rayleigh disc.

III. Results.

Values of K.—The procedure was to take at one sitting a number of observations of θ, the angular displacement, with known relative values of I_R and I_L, the intensities at the ears, and then subsequently to draw the straight line representing the mean of the observations and calculate

[1] Stewart and Hovda, Psych. Rev., XXV., No. 3, May, 1918, p. 242.

[2] The term "image" is here used as commonly in physics, *i.e.*, to refer to an apparent source of sound.

[3] Stewart and Hovda (loc. cit.).

the value of K. At least 24 to 50 observations were made for each curve, these being scattered so as to be "at random." In Table I. are shown the results thus far obtained, which, though not many, doubtless are sufficient to enable definite conclusions to be drawn.

TABLE I.

Frequency.	Observer.	Number of Curves.	Value of K.
256.....	S	3	16°
	B	5	30°
512.....	B	4	21°
	F	4	14°
	M	4	21°
1,024.....	B	8	10°
	F	8	7.8°
	M	8	18°

In Fig. 2 the straight lines represent the observations of B, the only experimenter who used all three frequencies. The observations of S are taken from the cited work of Stewart and Hovda, the 256 d.v. observations of B were taken by Mr. E. M. Berry in experiments conducted by himself and Mr. C. C. Bunch, the observations of B for the 512 and 1,024 d.v. were made by Mr. Berry in experiments conducted by Miss Caroline McGuire and the observations of F and M were also made with McGuire apparatus. In every case the linear logarithmic law seemed to hold Berry and Bunch showing that it held up to θ but slightly less than 90°. Two conclusions are to be drieved from the table, viz., that the constant K varies with individuals and that for an individual it decreases with increasing frequency.

Limits of the Application of the Logarithmic Law.—The first intimation that the law was not applicable to all ordinary frequencies with all individuals was found when the writer was unable at 1,024 d.v. to observe the rotation of the fused sound about the head with altered intensity ratios. This fact led to a brief series of experiments intended to ascertain at least some of the limitations of the applicability of the law. The apparatus, which is to be described in Part II. of this article, consisted essentially of a toothed wheel rotating in front of two bipolar telephone receivers. The currents produced actuated two head receivers and these, in turn, were attached in a suitable manner to stethoscope binaurals. With the apparatus adjusted for equal phase and intensity at the two ears, one of the rubber stethoscope tubes, right or left, was pinched, thus lessening the intensity on that side. Attention was confined to answering

the following questions, using a range of frequencies possible with the apparatus. Is there complete fusion and a rotation about the head of the image, apparently in accord with the logarithmic law? If partial fusion, where are the other images? Obviously, one could not test the law with such an apparatus quantitatively, but he could ascertain roughly at least some of the limitations of the applicability of the law. The purpose in these initial experiments was to find some of the limitations of the law and to point the way for future work. For the present purpose, then, it will be assumed that whenever a fused image moves from the median plane around to 90° right or left of that plane, the logarithmic law is followed. Clearly, when the image does move it is necessarily "fused" and hence in what follows the terms "fused image" or "complete fusion" or "incomplete fusion" imply an image moving as described. Only those frequencies were chosen which could be checked up by convenient tuning forks. Thus the frequencies 512, 640, 768, 896, 1,024, 1,280, 1,536, and 1,792 d.v. are the only ones referred to in this brief record.

With four observers *H*, *F*, *S* and *B* the following results were obtained. For *H*, with frequencies from 512 to 1,792 d.v. there was never complete fusion; from 640 d.v. to 1,536 there was no fusion; but below and above the two frequencies last named there was incomplete fusion. For *F*, up to 768 d.v. the fusion was complete, from 768 to and including 1,792 d.v. the fusion was incomplete. For *S* there is complete fusion up to 896 d.v., for 896 d.v. there was incomplete fusion, for 1,024 d.v. and 1,280 d.v. there was no fusion, and for 1,536 d.v. and 1,792 d.v. the fusion was practically complete. For *B* there were no frequencies possessing complete fusion, but there was a fused tone over the entire range and almost complete fusion at 1,792 d.v. These results are presented in Fig. 1, the two tones being represented by the single lines and fusion by the completeness of the connection between them.

When there was only partial fusion, one image did not rotate but remained directly in front, or, by trial, in the position determined by the difference of phase. It could, therefore, be ascribed to the phase difference effect. Thus the terms "partial" or "incomplete" fusion really mean fusion in more than one image. In the case where there was no fusion, and the tube at the left ear was being pinched the single image remained in front until the ratio of intensities was very large, *i.e.*, of the order of 200 with observer *S*, and then there appeared directly at the right, or approximately so, a second image which increased in clearness with further pinching, the image in front simultaneously disappearing, giving in the limit, as would be anticipated, but the one image to the right.

These experiments indicate that probably with most individuals (here four out of four) there is not complete fusion throughout all ordinary frequencies, but yet usually enough fusion to furnish a displaced image, that with some individuals (here two out of four) there are frequency gaps where the logarithmic law or any other law does not apply because of the absence of any fused image at all, and these gaps occur in the region having a frequency of the order of 1,000.

Another exception to the applicability of the law is the occasional serious deviation[1] from a straight line in attempting to represent a single individual's observations. Instead of a single straight line, required by the linear relation, the observations can, in these unusual cases, best be represented by two or three connecting straight lines of decidedly different slopes. The causes of such changes in the constant K at certain relative intensities are unknown.

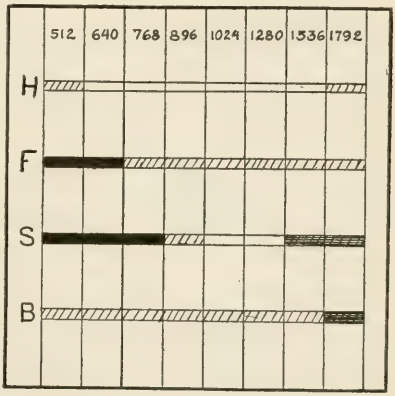

Fig. 1.

Intensity Combined with Phase Difference.—In actual localization there is a variation of both phase and intensity difference for a variation of position of the source, and it might be supposed that there might be found in the combination of the two factors an influence if the intensity factor were the only one. Yet such does not prove to be the fact. This was tested repeatedly, but not for all frequencies. Further reference will be made to these experiments in Part II.

IV. Theoretical Results.

The logarithmic law as shown above, is in harmony with experimental facts, at least as high as 1,042 d.v. with some individuals. We should, in order to be able to determine the importance of intensity in the binaural location of a pure tone, determine the relation between intensity and angular displacement existing in actual experience. With the present stage of development in sound measuring devices, we must resort to theory for the results. Assume the head to be rigid sphere 60 cm. in circumference and the ears diametrically opposite one another. The computations for the intensities at the two ears with the source at certain different distances and having certain different frequencies have already

[1] Referred to in Stewart and Hovda article, loc. cit.

been published.[1] We select from available data the distance 477 cm. as a common one, though at distances of this magnitude or greater there is but little difference in relative intensities. The computations apply to wave-lengths of 120 cm., 60 cm. and 30 cm. and these are frequencies of 287, 574, and 1,148 d.v. at room temperature.

If the comparison is to be made with experiment, the values must be plotted in the same manner. Inasmuch as the experimental values will

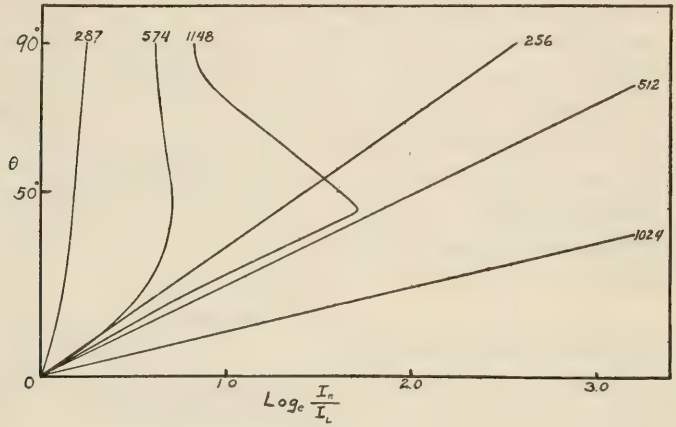

Fig. 2.

give straight lines only if θ and $\log_e I_R/I_L$ are used as coördinates, these are adopted in Fig. 2, the curved lines representing the computed values.

Discussion.

Comparison of Theory and Experiment.—In Fig. 2, the experimental curves are not extended to angles of 90° though the logarithmic law holds approximately to that displacement, but are shortened in order to show the theoretical curves clearly without using too large a figure. Thus, in making a comparison, the reader should extend these straight line curves to $\theta = 90°$. Clearly there exists neither quantitative nor qualitative agreement between the two curves for similar frequencies. In fact, the theory shows the existence of two values of θ for one value of relative intensity, whereas experiment shows only single values. The disagreement, quantitative and qualitative, is so very great that intensity cannot be an important factor in the location of a pure tone. Moreover, inasmuch as the intensity when combined with phase difference does not seem to have an importance not possessed by intensity when the phase difference is zero, we must conclude that for frequencies of 256 to 1,024 intensity can be only a minor factor.

It may occur to the reader to question the soundness of the conclusion

[1] Stewart, PHYS. REV., XXXIII., No. 6, 1911, p. 467.

because of the fact that the two ears may differ in sensitivity to sounds. But a consideration of the nature of the experiments and of the logarithmic law shows at once that minor deafness in one ear would not change the slope of the curves, or the value of K, but would merely displace the curve along the logarithmic axis.

Errors.—It is not advantageous to discuss sources of error in the apparatus itself when, as shown in the cited article by Stewart and Hovda, the individual errors are relatively large. But these errors in no way weaken the conclusion given above, for the divergence between theoretical and experimental values are too enormous. An objection may be made to the use of the stethoscope binaurals which were inserted in the ears thus closing the external meatus, a condition which does not obtain in actual binaural location. This objection is removed by the experiments in the article last mentioned wherein it is shown that binaurals inserted give the same effects as open binaurals which do not close the external meatus.

Intensity Effect with Complex Tones.—The conclusions of our paper refer to pure tones only. When complex tones are used there enters a difference in the quality at the ears occasioned by the variation of the ratio of intensity at the ears with frequency. The effect of this difference in quality in assisting in localization is unknown and should be ascertained by experimental investigation. Moreover, there is also an effect upon quality determined by the position of the source, *i.e.*, around a corner, behind a building, in another room, etc. That this effect upon quality assists in determining location is a matter of common experience. Hence intensity does become, indirectly, an important factor in the location of complex tones.

Weber's Law.—In the former paper already cited, the logarithmic law was presented as an extension of Weber's law. But it is difficult for the writer now to accept this position. Weber's Law seems to be a law of the nervous system, and, in its integrated form, states that the response is proportional to the logarithm of the stimulus. Now the nerve response in each ear is to a first approximation, independent of what transpires in the other. The apparent displacement, θ, is not the effect of either nerve response alone, but, in fact, is the form taken by the recognition of the differences between the nature of these two nerve responses and hence is not at all a law concerning nerve response and stimulus as is Weber's Law.

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THE FUNCTION OF INTENSITY AND PHASE IN THE BINAURAL LOCATION OF PURE TONES. II.

BY G. W. STEWART.

PART II. PHASE.

SYNOPSIS.

Linear Relation; Phase Difference and Apparent Angular Displacement of the Source of Sound.—That a linear relation exists between phase difference at the ears and apparent angular displacement from the median plane between them, is demonstrated for pure tones within the range 100 to 1,200 d.v. Extended experiments are made with two individuals but additional evidence is adduced to show that the results have a general applicability.

Linear Relation; the Variation of the "Constant" with Frequency.—The "constant" in the above linear relation, or the slope of the curve with the two variables as coordinates, is approximately a linear function of the frequency.

Upper Limit of the Phase Difference Effect.—The upper limit of the phase difference effect seems to be from 1,000 to 1,500 d.v.

Double Images.—It is shown that the phase difference effect, if it be the controlling factor in localization, may produce multiple apparent sources. For a frequency of 1,024 d.v., the double phantom sources are found in actual experiment to have the relative positions indicated by the phase effect.

Intensity-effect and Phase-effect Combined.—It is found that when intensity difference and phase difference at the ears exist simultaneously, neither has a greater effect than when applied separately. Hence quantitative results obtained by using intensity difference only or phase difference only are transferable to the case of the combination.

Direct Perception of Phase.—Phase effect cannot be explained by a direct or by an indirect effect of intensity, for the phase effect at certain frequencies and with some individuals may exist when the intensity effect is wholly absent. Direct perception of phase is thus evident.

Theoretical Computation and Comparison with Experiment.—The phase difference at the ears for various angular displacements of the source from the median plane are computed for three frequencies scattered over the range under consideration. The relationship is not strictly linear but very roughly so. A comparison is made of the theoretical and experimental ratios of angular displacement and phase difference. There is a satisfactory quantitative agreement.

Phase Difference the Most Important Factor in Localization.—The significance of the quantitative agreement of experimental and theoretical results is presented and the conclusion derived that phase difference is the most important factor in localization of the source of a pure tone, 100 to 1,200 d.v. situated in a horizontal plane in front of the observer and 90° either side of the median plane. This conclusion is extended to include the region behind as well as in front of the observer. There are more factors in actual localization than in the case assumed in the theoretical discussion; for example, reflection and alterations in quality are doubtless important factors in the general case. Above the limit of the phase difference phenomenon, viz., approximately 1,200 d.v., intensity must become an important factor, but its importance varies with the individual.

I. INTRODUCTION.

THAT phase difference at the ears affects the localization of the sound source, has been known[1] for a number of years. Recently Hartley[2] has produced evidence in favor of the conclusion that the phase difference of a pure tone at the ears is the controlling factor in the location of such a sound source by an observer who holds his head stationary. The present paper will present a considerable addition to this evidence, making the conclusion just stated a fact to be accepted with full confidence.

In order that correct conclusions may be obtained in the problem of localization it is important that the simplest elements be involved in the first approximations. Thus the writer has used tones which were reasonably pure and has sought to ascertain not all the factors in localization, but rather the one clearly the most important.

II. APPARATUS.

The object of the experiments was to determine for various frequencies the relationship between the phase difference at the ears and the corresponding apparent angular displacement from the median plane. The observations were in part taken with a source of sound which for simplicity will be called a "phaser." The tones were not pure, but the results have been checked by additional experiments with tuning forks, as hereinafter described, and the use of the phaser may thus be considered as satisfactory for the purposes of these experiments.

The "Phaser."—This instrument consisted essentially of a rotating toothed wheel with two telephone bipolar receiver magnets placed radially close to the teeth and capable of being separated from each other by a variable known number of degrees of rotation of the wheel. Currents of the same frequency and complexity were incuded in the two bipolar receiver magnets, each of which was connected to a head receiver, and the phase difference of the currents could be controlled by the separation of the two bipolar receivers around the circumference of the rotating wheel. Thus the phase difference corresponding to any apparent angular displacement could be secured. A brief description of the phaser will suffice, for a study was not made to ascertain the shape of tooth giving the purest tone attainable or otherwise to produce the best instrument for the purpose. The motor was a small series one, designed for a rotary spark gap of a wireless set, and capable of operating continuously at 6,000 r.p.m. For securing fairly constant speed the field and armature

[1] For a review of literature see Stewart, PHYS. REV., Vol. IX., June, 1917, p. 502.

[2] Hartley, PHYS. REV., Vol. XIII., June, 1919, p. 373.

were operated on independent storage batteries. Toothed wheels of two, four, six, eight and twenty projecting teeth were used. The one with two teeth consisted, in fact, of a bar of iron approximately $9 \times 2 \times 0.6$ cm. The tone given by such a wheel is surprisingly pure and presumably so because the tones other than the fundamental are necessarily made up of intermittent vibrations and, as is well known, such intermittent tones are not readily recognized by the ear. For the same reason the tones produced by each of the other wheels were purer than one might anticipate.

By using in each circuit of the bipolar and head receiver an impedance having high self induction, the overtones were cut down relatively more than the fundamental. In the twenty-teeth wheel at high speed there occurred a tone having the frequency of the rotations per second of the wheel. This was practically eliminated by using a small capacity in series. But the excellent check on the phaser experiments with tones from tuning forks will be sufficient evidence that the results of the experiments are reliable and may be, so far as present accuracy is concerned, considered to be those obtained with pure tones.

Throughout the experiments with the phaser, in order to insure a sufficiently constant sound conductivity from each head receiver to the ear, stethoscope binaurals were inserted in the ears and were connected by rubber tubing to brass cylindrical plugs which fitted the openings in the head receivers.

The Circular Scale.—The observer sat at the center of a scale, 170 cm. in radius, prominently marked at 5° intervals up to 90° on each side of the zero which was directly in "front," or in the median plane between the ears. The scale was mounted at a convenient level, about that of the chin.

Tuning-fork-time-test.—Sounds from two electrically actuated tuning forks, slightly different in frequency, were led one to each ear by rubber tubing and stethoscope binaurals. As described in a former paper,[1] the apparent location of the sound rotates about the head with the changing of the phase difference of the two tones. By recording on a chronograph the time required for a rotation of the image[2] through an angle and comparing that with the time of a complete beat or phase difference variation of 360°, the phase difference corresponding to the apparent angular displacement from the median plane could be secured.

Tuning-fork-phase-difference-test.—In this test electrically driven tuning forks were used. The apparatus is that described by Miss M. Simpson.[3]

[1] Stewart, loc. cit.

[2] The word "image" is here used as in physics: it is the apparent source of the sound.

[3] Simpson, PHYS. REV.

With it settings of phase differences could be made at will and thus the relationship between these differences and the apparent angular displacements from the median plane could be ascertained.

III. Experimental Results.

Linear Relation.—That the apparent angular displacement from the median plane, θ, is a function of φ, the phase difference at the ears, was already well known, but the relationship between θ and φ was to be ascertained. This was accomplished as follows: The observer sat at the center of the scale and listened to the sound from the phaser. The assistant closed the circuits for two to five seconds and then opened them for one or two seconds' rest, repeating the operation until the observer signalled that he had determined the apparent location of the fused sound. Settings of the phaser were chosen so as to make the observations "at random." Fig. 1 shows the general nature of the results always obtained not only with the phaser but also with the tuning fork phase difference apparatus mentioned above. It is clear from Fig. 1 that a

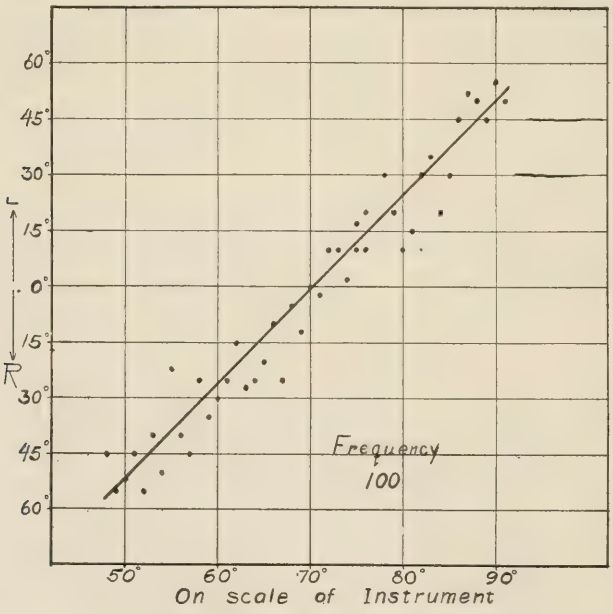

Fig. 1.

straight line will represent the mean of the observations more satisfactorily than any other curve. This was found to be the case at all frequencies. Hence the experiments were planned so as to determine merely the ratio φ/θ for each frequency, and this was accomplished by

plotting a curve from each observational "run" consisting of 16 to 20 observations. Since the completion of these observations another method of procedure, equally fair, has been found to produce much less individual error than shown in Fig. 1.

Values of φ/θ.—The values of φ/θ, or the ratios of phase difference to apparent angular displacement, are shown in Fig. 2.

The dots represent the values obtained from experimental curves

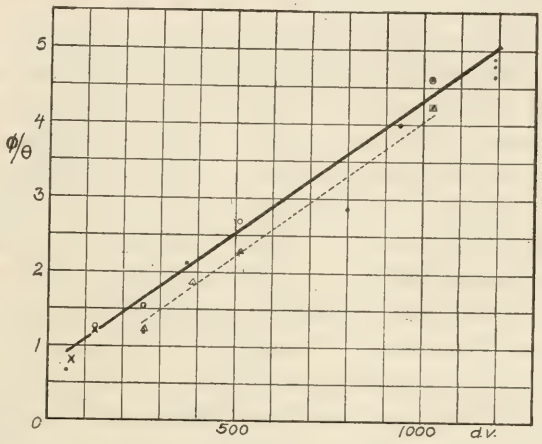

Fig. 2.

similar to that of Fig. 1, each involving 16 observations, the method being the use of the phaser with the additional aid of Helmholtz resonators presented to the head receivers and connected to rubber tubing leading to the stethoscope binaurals.

The circle with a dot center is the average of the values of φ/θ taken from four curves, the observations being made without resonators but with great care to secure a tone as nearly like that of a tuning fork as practicable by the use of inductance. Such a tone was not difficult to obtain for the tubber tubing caused rapid absorption of the overtones of this frequency of 1,024 d.v.

The crosses represent the observations taken by means of timing the rotation, the one for 64 d.v. being the average of 61 observations, and for 128 d.v. the average of 81 observations.

The small circles represent the average of curves similar to that of Fig. 1 obtained in the tuning fork-phase-difference-test. In fact, they are the observations reported by Miss Simpson.[1] The number of curves for the 128,256 and 512 d.v. were eight, seven and nine respectively.

The deltas represent observations taken by a second observer *F*,

[1] Loc. cit.

using the phaser, each point being the average value of the ratio taken from five curves. The method of observing was modified, the experimenter adjusting the phaser at will by a mechanical connection. The observations became more consistent without any other error being introduced.

It should be here recalled[1] that as the phase changes the sound travels about the head from the median plane in front and external to the head to the side leading in phase, then approaches and enters the ear, passing through the head quickly at 180°, then to the other ear and from thence around to the front along a path symmetrical to the one just described. This seems to be the movement of the image, but in the paper last cited the frequency used did not reach 500 d.v. The curve now shown, Fig. 2, shows that this complete movement is impossible for the higher frequencies. For, with the frequency of 1,024 d.v. the ratio of φ/θ is 4.4 which means that when φ is 180°, θ has become 41°, thus the image does not pass to the side of the head but, at $\varphi = 180°$, jumps from $+ 41°$ to $- 41°$, and then proceeds to 360° or 0° in a path symmetrical to the one from 0° to $+ 41°$. This jump has already been mentioned by Bowlker.[2] The fact that two different observers should obtain similar curves indicates the generality of the nature of the results. In all, four observers were tested for the extreme limits of the frequencies. The values of φ/θ for the four at 1,024 are 4.6, 4.5, 5.4 and 4.2. At the lower limit they were likewise in reasonably close agreement. Thus it is obvious similar curves would be had in general. One observer, *M*, a fifth, did not have a similar curve, the values obtained being reported by Miss Simpson.[3] But this observer was unusual in the effect of phase difference, being unable to get the image located external to the head.

There seems thus to be no reason to doubt the general conclusions from Fig. 2, though individual exceptions will be found. It should be pointed out that experiments thus far made indicate that the curve for φ/θ may bend sharply toward the origin at lower frequencies, but this region remains to be studied carefully.

The Frequency Limit.—By means of the phaser, observer *S* found that at 1,280 d.v. the phase difference continued to give an apparent displacement of the source, but that at 1,536 d.v. the source did not seem to move. All the observers tried, four in number, were found to have the same experience. Probably, then, this phenomenon ceases at from 1,000 to 1,500 d.v. with individuals generally. More[4] states that with

[1] Stewart, PHYS. REV., Vol. IX., June, 1917, p. 502.

[2] Bowlker, Phil. Mag. (6), XV., 1908, p. 318.

[3] Loc. cit.

[4] More, Phil. Mag., XVIII., 1909, p. 308.

1,024 d.v. his judgment became untrustworthy and that with 3,000 the effect ceased entirely. Rayleigh[1] gives his limit as 768 d.v. Myers and Wilson[2] state that with high frequencies the effect ceases. Preliminary experiments have shown the writer that the effect does not recur at frequencies of 2,000, 3,000, and 3,500 d.v.

Double Images.—It will be noted that, as already described, for 1,024 d.v., when φ has reached 180°, θ has become 41°. Experimentally, at 180° one should then have two images, + 41° and − 41°; and this was found to be the case. But if φ is made greater than 180°, will there continue to be two images, one being between $\theta = 0°$ and $\theta = -41°$, and the other between $\theta = +41°$ and $\theta = +90°$? Such was found to be the fact, but the distinctness of the second image differs with the observer.

In view of the above result a test was made in the open as follows: A blind-folded observer seated himself upon the ground and located without rotating his head, an electrically driven tuning fork, mounted on a resonator and situated on the ground at selected points of a semicircle of five meters radius. Experiments were conducted in such a manner that the only assisting sound was the pure tone of the tuning fork. For example, during transport from one point to another, the electromagnet operating the fork was short-circuited and the observer covered his ears. The fork had 1,024 d.v. and was driven in tandem by an electrically driven 512 d.v. tuning fork. The ground was heavily grassed, the observer held his body and head strictly stationary; the source was placed close to the ground and every precaution taken to avoid the influence of reflection. The typical results obtained are shown in Table I. A similar experiment by observer *H*, gave an average of 96° and, by observer *F*, a 90° separation. Observer *F* found a similar average with the phaser of 88°. In the 16 observations in Table I., there were four cases where the observer was incorrect in his impression as to which of the two locations was actually that of the source.

The significance of these outdoor experiments will appear in a later portion of this paper.

Combined Effect of Phase and Intensity.—In Part I. of this report was discussed the experimental effect of intensity only and in Part II. the chief variable is phase only. If either intensity or phase has any additional effect when in combination with the other, this should be ascertained. The following method was adopted. The phaser was set for a frequency of 512 d.v. At this frequency, according to the theory referred to in Part I., the ratio of intensities at the ears would be 0.6

[1] Rayleigh, Phil. Mag., XIII., 1907, p. 214.

[2] Myers & Wilson, Proc. Roy. Soc., LXXX., 1908, p. 260.

TABLE I.

Position of Fork.	Position of Image at Right of Front.	Position of Image at Left of Front.	Separation of Images.
40° L	20°	50°	70°
80° L	10°	80°	90°
50° L	20°	60°	80°
20° L	—	15°	—
30° R	20°	70°	90°
60° R	60°	50°	110°
90° R	90°	10°	100°
10° R	0°	0°	—
70° R	65°	30°	95°
0°	10°	—	—
30° L	45°	40°	95°
90° L	15°	80°	95°
50° L	15°	70°	85°
10° L	—	5°	—
40° R	40°	40°	80°
60° R	50°	50°	100°
			Av. 91°

for a displacement of 90° from the median plane. Using the experimental value of $K = 14°$, for observer F at approximately this frequency, it is found that a ratio of 0.6 would actually produce a displacement of 7.5° from the median plane. One of the tubes leading to the ears was pinched so that a displacement of approximately this amount was produced. Then a series of observations with varying phase difference was made with this selected intensity ratio. If intensity had any peculiar effect when in combination with phase, then it would be expected that a curve between phase difference and angular displacement would not be the same as that with equal intensity. But experiment showed that the same curve was always produced, though of course shifted on the θ axis by the same amount at every point. In other words, a difference in intensity merely caused a constant shift in the displacement θ. This experiment was repeatedly tried with differences of intensity causing a shift as great as 15° and yet without altering the nature of the phase difference curve. The ratio φ/θ was unchanged in every case. Thus neither intensity nor phase has any peculiar effect when in combination with the other and our results for either acting alone are to be considered as correct in all cases with pure tones. Thus although one may produce at the ears the same intensity ratio and phase difference as that occurring in actual localization, the effect of intensity is practically nil and the location is determined chiefly by phase difference.

IV. Theoretical Considerations.

φ a function of θ.—The results of computations from theory of the phase difference at the ears with the source in a horizontal plane and at an angle of θ from the median plane, were first presented by the writer.[1] For clearness the original curve is herewith reproduced as Fig. 3.

The ordinates, θ, refer to the angular displacement of the source of

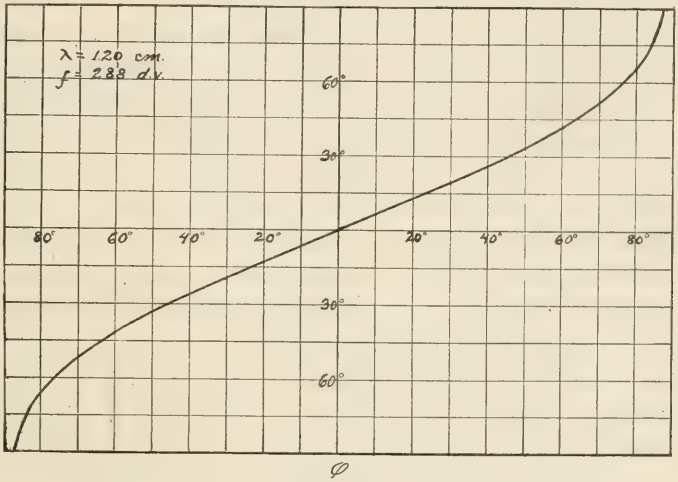

Fig. 3.

sound from the median plane, and the abscissæ, φ, refer to the phase difference at the ears. The computations assume the source at a distance of 477 cm., the wave-length 120 cm., the head a rigid sphere 60 cm. in circumference, and the ears diametrically opposite. The similarity of the curve to that obtained experimentally as in Fig. 1, is obvious.

Values of φ/θ.—The question arises as to whether or not the values of φ/θ for the theoretical case of the source actually displaced at an angle θ, correspond to the experimental values where φ is produced and θ chosen. If the values are in agreement, it would seem that the cases are similar, and this means that in the location of a sound source without head rotation, the phase difference is the important factor.

In order to compare the theoretical and experimental values, certain approximations were made. The experimental curve is linear so far as observations could determine. The straight line drawn was influenced largely by the observations of θ having a mean of 25°. Hence the theoretical values of φ/θ to be used in comparison should be those for θ about 25°. These values do not differ from $d\varphi/d\theta$ by more than nine per cent. on the average.

[1] Stewart, Phys. Rev., N. S., Vol. IV., September, 1914, Fig. 3.

The theoretical values were obtained from the results published in the article last cited and from the curves of Hartley[1] who assumes the circumference of the head 55 cm. and the ears 165° apart and the source of sound at a great distance. They are plotted as indicated in Fig. 4. The curves drawn are those of Fig. 2 and represent the experimental results. It is obvious that the theoretical and experimental values correspond remarkably well, the nature of the results being considered.

Intensity Effect.—In Part I. of this report the results of a study of the intensity effect, with frequencies 256 to 1,024 d.v., have been presented and the conclusion definitely drawn that intensity differences at the ears cannot be important factors in localization over this range of frequency.

V. Discussion and Conclusion.

Significance of Fig. 4.—Two obvious factors in localization are found in the differences in phase and intensity at the ears. The remarkable quantitative agreement between theory and experiment shown in Fig. 4

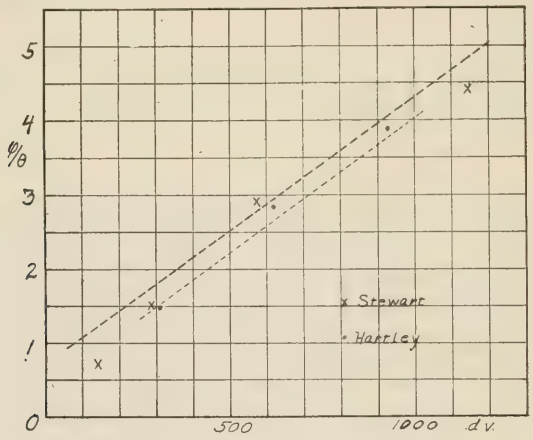

Fig. 4.

has the following significance: the angular displacement from the median plane produced by a given phase difference at the ears proves to be the very position for the source of sound that will, theoretically, produce the aforesaid phase difference at the ears. This quantitative agreement, although approximate, is highly satisfactory if appropriate allowance is made for the nature of the experiments. The conclusion must be that the phase difference is a factor of high importance in the frequency range considered.

The experiments of Bowlker[2] are in close agreement with Fig. 2. He

[1] Hartley, Phys. Rev., Vol. XIII., June, 1919, p. 373.

[2] Bowlker, loc. cit.

found that a wave-length of 36 inches was the one giving a displaced image of 90° for a difference of 180°. This would be a frequency of about 380 d.v. Reference to our curve, Fig. 2, shows that the frequency in our experiments would be 352 d.v. for one observer and 440 d.v. for the other. Bowlker used organ pipes for the source and hearing tubes $2\frac{1}{4}$ inches in diameter. With such a narrow aperture the relative intensities in the two tubes, one to each ear, would be fairly independent of the directions of the axes, and thus he was dealing with practically equal intensities as in our experiments.

Inasmuch as in Part I. of this report, intensity was found not to be an important factor in the binaural location of pure tones, our conclusion as the high importance of phase, is given additional weight.

Extension of Limited Application of Experimental Curve.—The results as presented in Fig. 2, do not cover all values of displacement either side of the median plane, but strictly speaking only up to the values of displacement for which the phase difference is 180°. Does phase difference remain the most important factor for these higher frequencies, where, as we have shown there is more than one image? Since the multiple images are found both with the phaser and in the open in actual localization, the additional factor of intensity entering the latter case does not modify or alter the effect of phase and hence its importance. In other words, phase difference remains the most important factor in localization throughout the range of frequency considered and for displacements of 90° from the median plane. Moreover, computations from the theoretical values of Hartley and of the author already referred to show that the separation of the images for 1,024 d.v. is approximately that obtained in the open, viz., 90°.

The evidence in hand both experimental and theoretical seems to establish beyond reasonable doubt that localization in the horizontal[1] plane for pure tones of 100 to 1,200 d.v. depends almost entirely upon phase difference at the ears. But the above discussion refers to localization within the region 90° on either side of the median plane and in front only. This first approximation in the explanation of localization need not be considered as limited to the front region only, for the well known confusion between locations in the front and in the rear indicates that the discrimination between such positions symmetrical to the median plane, rests upon additional factors probably of secondary magnitude. Indeed, many observers find that the difference of phase at the ears causes them

[1] All of our experiments refer to the horizontal plane, but a note should be made of the fact that in the experiments with the phaser and the forks the rotation was usually not strictly in a horizontal plane but could be made so by a slight forward tilt of the head.

to locate the source in the rear rather than in front. For the present, then, we are to assume that the most important factor in the frontal region is also the most important factor in the localization in the rear, leaving for future consideration the factors that distinguish between the positions symmetrical with the line connecting the ears. It is to be admitted freely, then, that the present paper does not offer a complete explanation of localization, but rather proves that the difference of phase phenomenon is the most important single factor in sound localization of a pure tone in a horizontal plane.

Complexity of Factors in Actual Localization.—One might decide that, since the only physical factors in a pure tone of given frequency are phase and intensity, and since we have all but eliminated intensity as a factor in localization, the only important factor left is that of phase difference. But this cannot be true in the sense that the phase difference is that produced merely by a single source and diffraction about the head as a sphere. For there are always present reflecting surfaces which are extensive enough to produce images. This is especially true of frequencies of the order of 1,000, having a wave-length of 34 cm. Although the effect at the opening of the external meatus is still expressible in phase and intensity, yet, in contrast to the case of a simple source, we have, in general, the equivalent of several sources, with most of them on the same side of the median plane as the source. Now while it has not been demonstrated that the two ears have a "resolving power," yet there would result from reflections an apparently diffused source of sound instead of the original source only. Consequently the observer could distinguish between a location on the right and left side of the median plane. Thus, assuming that phase difference is the most important factor in localization, it by no means follows that the case is as simple or needs to be as simple as that of a source and a rigid sphere with the two ears located diametrically thereon. The complexity of conditions involving reflection gives the single factor, phase difference, a greater opportunity to secure accurate location than die the simple theoretical case exist.

That reflecting surfaces are important was readily shown by the effect of rotating the head when attempting to locate the source in the open air experiments described. A slight rotation would sometimes shift the apparent location by as much as 20°. Doubtless there are other factors which enter into the localization of a pure tone. There are at least two additional factors in the case of a complex tone; they are the difference in quality at the two ears produced by diffraction, and the difference in quality of the sound arriving, depending upon the location of the source.

Intensity a Factor.—If the frequency exceeds the limit of phase difference sensitivity, our chief factor, phase difference, ceases to exist and we are dependent upon intensity effects, including not only the intensities produced at the ears by the source, but also by the mirror images. As elsewhere recorded, the writer has found that the effect of intensity-ratio as embodied in the formula presented earlier in Part I. of this article, becomes seriously modified in a limited region of frequency. With some individuals, for this frequency band, there is no displacement law as above given. For others, there ceases to be complete fusion in this region, though it is possible to locate a fused tone as well as to recognize another image at the same time. But it is not the purpose of the present article to cover the region above 1,200 d.v. as this will be left for further experimentation.

The Ear as a Physical Instrument.—Physicists are interested in the ear as a physical instrument and one might well inquire as to whether or not the experimental curve here shown can indicate any mechanical requirement of the ear. If the curve had been a straight line passing through the origin, it might then be said that, since equal time intervals correspond to phase differences that are proportional to frequencies, and since φ/θ would then be proportional to frequency, therefore equal values of θ would be obtained by equal time intervals. But the curve, if linear, does not pass through the origin and equal time intervals correspond approximately to equal angular displacements only at the higher frequencies. Hence we cannot say the angular displacement is caused simply by a time interval. It is more reasonable to assume that the phenomenon we are discussing is not one of the ear regarded merely as a mechanical instrument, but of the entire phenomenon of hearing.

Phase Difference Phenomenon Independent of Bone Conduction.—In binaural beats with a frequency of from 40 to 400 there is a complete rotation around and through the head of the fused sound as φ changes from 0° to 360°. This has been fully discussed and explanations offered,[1] but one writer[2] has interpreted this explanation as requiring cross-conduction for the phenomenon of localization of sound. This is incorrect and an additional word is necessary. As the fused sound rotates from $\theta = -90°$, through $\theta = 0$, to $\theta = 90°$, the location of the image is external to the head just as in the usual localization of a source of sound. But during much of the other half of the cycle the image is within the head and the experience is wholly different than in ordinary localization. The writer did use cross-conduction to explain this internal half of the

[1] Stewart, PHYS. REV., Vol. IX., 1917, p. 514.

[2] Hartley, loc. cit.

cycle but in no wise does this mean that cross-conduction enters into the act of localization of sound. In fact, as already indicated, we are interested only in the portion of the rotation $\theta = -90°$ to $\theta = +90°$, anticipating that the chief factor in localization in this region will prove to be the chief factor in general.

Perception of Phase.—As has been mentioned in various articles referring to perception of phase, many psychologists assert that the response of a sensory nerve is independent of the nature of the stimulus and if this is true, phase relations could not be found in the nervous impulses. Thus direct perception of phase could not exist. It would seem that the simplest interpretation of our experiments is the direct perception of phase. An effort has been made by some to explain the apparent perception of phase by the indirect effect of intensity arising from bone conduction between the ears. It would seem that the intensity effect is too small to warrant such an explanation. But what appears to be a crucial experimental test has been found. As recorded in Part I., observers *H* and *S* could not get any intensity effect whatever at, 1024 d.v., yet the same observers responded to phase difference at this frequency without any hesitancy. Thus, the effect of phase difference cannot be explained by any indirect intensity effect. It would thus seem necessary to admit direct perception of phase, at least until a possible different interpretation of the effect of phase is secured.

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EXPERIMENTS IN BINAURAL PHASE DIFFERENCE EFFECT WITH PURE TONES.

BY MARGERY SIMPSON.

SYNOPSIS.

There are no published results which show the variation of angular displacement of the apparent source of sound with changes in phase difference at the ears.

Linear Relation between Angular Displacement and Phase Difference.—Tuning forks were used for the source of pure tones. Equal intensity was maintained at the two ears, while a difference in phase was secured by a difference in the lengths of the sound paths to the two ears. Results were obtained which are best represented by a linear relation between the phase difference and the angular displacement of the sound image or phantom from the median plane. The ratio of phase difference to apparent angular displacement varied with the frequency. A comparison of these ratios was made with the theoretical values as calculated by Stewart and Hartley, but the frequency range was not sufficient to enable final conclusions to be drawn.

THE purpose of this article is to give the results of certain quantitative measurements of the angle through which the sound image or phantom, as termed by psychologists, is displaced from the median plane for a given difference of phase in the sound at the two ears.

Method.—Tuning forks were used as the source of pure tones. The given difference of phase was secured and maintained by means of unequal lengths of sound paths to the two ears, while the intensities at the two ears were kept equal.

Apparatus.—The source fork was driven in tandem with an electrically maintained driving fork of the same frequency. The sound was conducted from the source fork through two brass tubes which were rigidly fixed to the iron mount of the source fork, and whose axes were perpendicular to the face of the vibrating fork. One of these, a telescoping tube, provided for the intensity adjustment, and carried a vernier sliding upon a millimeter scale. The driving fork and the source fork were each inclosed in wooden boxes, heavily lined with hair felt which served as sound screens. The sound was led from the brass tubes through equal lengths of rubber tubing to a second pair of brass tubes. One of these had a fixed length, and the other, a telescoping tube, provided for the phase adjustment and carried an indicator sliding upon a millimeter scale. A brass shut-off and reversing valve was attached to the ends of the tubes

last named. The sound was then conducted through equal lengths of rubber tubing and through stethoscope binaurals to the ears.

It is obvious that changing the length of one of the paths would, through changes in absorption and especially in resonance, produce a difference of intensity at the two ears. Hence, a calibration for equal intensity at the two ears was necessary. This was made for the frequencies 512 and 256 d.v. by means of Rayleigh discs. No Rayleigh disc was available for the 128 frequency. But inasmuch as experience with the 256 and 512 frequencies showed that the calibration was a refinement not warranted, on account of the inaccuracy of the observations and the very small error due to intensity differences, the more crude method of adjusting by the ear was employed. When the two intensities with equal phase produced no noticeable shift of the sound image from the median plane, the two were regarded as equal.

This method of determining the phase difference depends upon the velocity of sound. As the velocity is diminished in passing through tubes, it was calculated for the three frequencies, assuming the correctness of Helmholtz's[1] theoretical equation and using Seebeck's experimental value for the arbitrary constant. These considerations gave 339.4 meters per second for the velocity at room temperature. From this value conversion factors were obtained which express the degrees phase difference per centimeter of path.

Method of Observation.—A series of settings for varying phase differences, leading in phase at either ear, was given at random to an observer. The observer, who was seated at the center of a graduated arc 65 cm. in radius, localized the sound image in degrees right or left of the median plane. The operator opened and closed the shut-off valve at the speed desired by the observer, and continued this until the sound image was localized. The greatest difference in path was about one fourth wavelength, and the phase was made to lead in either ear by the reversing valve.

Results.—Experience with this apparatus showed that if a number of observations are made and plotted with the phase difference, ϕ, and the angular displacement, θ, as coördinates, the mean of the observations was best represented by a straight line passing through the origin. The experiment was conducted with the same four observers for each of the three frequencies 512, 256, and 128 d.v. Three of the observers described the image as distant; that is external to the head, and the fourth localized it as within the head and projected it upon the arc. This localization within the head seems unusual and undoubtedly lessens the reliability

[1] Barton, Text Book on Sound, p. 542.

of the results of this observer which show greater disagreement than those of the other three.

The image seemed most sharply defined for the 512 frequency. With the lower frequency the image was broad and especially so in the region of maximum displacement. The slopes of the curves mentioned were

TABLE I.

Observer.	$N=512$.		$N=256$.		$N=128$.	
	Number of Curves.	ϕ/θ.	Number of Curves.	ϕ/θ.	Number of Curves.	ϕ/θ.
S.....	9	2.69	7	1.55	8	1.27
H.....	5	2.15	2	1.09	1	.88
B.....	9	1.87	12	1.61	16	1.24
M.....	10	1.26	16	1.16	16	.97

determined and these values of ϕ/θ are reported in Table I. These experimental values of ϕ/θ for each observer are plotted as a function of the frequency. (Fig. 1.)

Discussion of Rresults.—The results obtained through the range of frequency 128 to 512 d.v. are in general agreement with those of previous

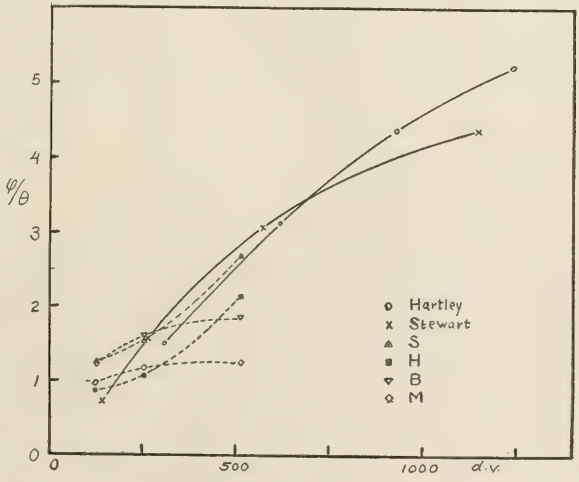

Fig. 1.

qualitative work as summed up by Stewart:[1] that phase difference in sound at the ears gives rise to a sense of localization; and that the phase difference necessary to produce a given displacement increases with the frequency.

Stewart[2] has calculated theoretically the phase difference at the ears

[1] G. W. Stewart, PHYS. REV., 9, 502, 1910.

[2] G. W. Stewart, PHYS. REV., 33, 467, 1911.

for varying positions of the source upon certain assumptions. He gives these calculated values for the frequencies 276, 553, and 1,106 d.v. and also plots a curve for the frequency 276 d.v.

Hartley[1] has extended these calculations to the frequencies 310, 620, 930, 12,40, and 1,860 d.v. His calculations are based upon assumptions slightly different from those of Stewart's.

These theoretical curves show that as the frequency increases, the phase difference necessary to produce a given displacement increases. The number of degrees phase difference necessary to produce a degree shift in localization for the various frequencies was determined from the theoretical curves by finding the slope through the origin. These values of ϕ/θ are plotted as a function of the frequency on the same graph as the experimental values. The close agreement of the theoretical and experimental results indicate that phase difference is an important factor in the localization of sound, at least within this range of frequencies.

The difference in the slopes obtained experimentally for the individual observers for each frequency indicates individual differences in the degree in which phase difference gives rise to localization. The value of ϕ/θ for each observer increases as the frequency increases, but not in the same ratio. The slopes for the individual observers varied from day to day and during the day, perhaps due to his physical condition.

These quantitative results would seem to support the direct perception of phase. But these results must be extended to other frequencies before any definite conclusion can be reached as to the exact rôle which phase difference plays in the localization of sound.

Summary.—This quantitative experimental work deals with the localization of the image of a pure tone with equal intensity and a phase difference at the ears. The results confirm in general those of previous investigators, within the range of frequencies 128 to 512 d.v., and may be stated as follows: phase difference gives rise to a sense of localization; phase difference is an important factor in localization; the value of ϕ/θ is a function of the frequency. The experimental values of ϕ/θ are in good agreement with the theoretical results of Stewart and Hartley.

I wish to express my acknowledgments to the members of the physics staff, and especially to Professor G. W. Stewart who suggested the problem and who acted as observer; to Mr. E. M. Berry and Mr. H. M. Halverson who acted as observers and to Mr. J. B. Dempster for his assistance in the construction work.

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[1] R. V. L. Hartley, PHYS. REV., 12, 373, 1919.

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[Contribution from the Physics Laboratory, State University of Iowa.]

THE VAPOR PRESSURE CURVES OF SOLID AND LIQUID SELENIUM NEAR THE MELTING POINT.

By L. E. Dodd.

Object and Scope.

The object of this investigation has been to determine by the method of molecular flow the saturated vapor pressure of selenium in the vicinity of the melting point. This required the determination of both the sublimation curve of the crystalline solid and the vaporization curve of the liquid. Further, the aim has been to find from these data values for some of the physical constants of selenium near its melting point.

Molecular Flow of Gases and Knudsen's Equations.

Knudsen's[1] equations for resistance to molecular flow of gases are, for an aperture in a thin partition,

$$W_1 = \sqrt{2\pi}/A_1 \qquad (1)$$

and for a tube

$$W_2 = {}^3/_8\sqrt{\pi/2}\int^L O/A_2^2\, dl \qquad (2)$$

where l is length of tube, O is circumference, and A_2 cross-sectional area, which may be variable.

[1] Knudsen, *Ann. Physik.*, 28, 75, 999 (1909).

For a combination of a thin partition with an aperture placed across a tube, and the tube, the total resistance is $W = W_1 + W_2$. If there is a steady flow of gas through this combination, such that the pressure where the gas enters the tube is p_2 and the pressure where it issues is p_1, the quantity flowing through in time t is given by

$$G = 1/W \cdot (p_2 - p_1) \sqrt{\rho_1}\, t, \qquad (3)$$

where G is weight of gas in grams, and ρ_1 the density at unit pressure and at the temperature of the apparatus. All quantities are in absolute units. ρ_1 can be obtained from the known density of the gas at unit pressure and 0°,

$$\rho_1 = (\rho_1)_0/(1 + at), \qquad (4)$$

where a is the expansion coefficient and t the temperature. Knudsen verified (1), (2) and (3) experimentally with the gases hydrogen, oxygen and carbon dioxide.

Application of the Method of Molecular Flow to the Measurement of Saturated Vapor Pressures.

Knudsen[1] used the method of molecular flow to measure the vapor pressure of mercury at lower temperatures than are possible by other methods. For calculation of ρ_1 in this case he employed (4), which is applicable to any substance in the vapor state if the number of atoms per vapor molecule is known, since

$$(\rho_1)_0 = (1409.2 \times 10^{-12})/(2 \times 16)nA, \qquad (4a)$$

where the number in the enumerator is the density of oxygen at unit pressure and 0°, n an integer expressing the number of atoms per molecule, and A the atomic weight. He made 2 standard determinations of mercury vapor pressure at 0° and 7°, respectively The vapor was condensed at the temperature of solid carbon dioxide, so that p_1 was negligible. For the other temperatures the vapor was condensed at 0°, and the standard value already found was used in this case for p_1.

Following up this application of the method Egerton,[2] at the suggestion of Nernst, applied it to determine the vapor pressure of metallic zinc and cadmium. To save time in these pressure measurements, Egerton decreased W in (1) by using, instead of one, several circular apertures in parallel in the same thin partition. He showed that the total resistance of a partition varies inversely as the number of apertures. Egerton used as many as 17 apertures in parallel with good results.

The present work was a continuation of some preliminary experiments by F. C. Brown, applying this method to selenium. As far as known to the present writer, this is the first application of the method to a substance with other than a monatomic vapor at the temperatures of the ex-

[1] Knudsen, *Ann. Physik*, **29**, 179 (1909).

[2] Egerton, *Phil. Mag.*, **33**, 33 (1917).

periment. The preliminary results were spoiled by the lack of a high enough vacuum and by an important temperature difference inside and outside the vapor tube. In the present investigation selenium crystals of large size, as first produced by Brown[1] and furnished by him for this work, were used for the element in the solid state.

Apparatus.

The Egerton type of apparatus, which with some changes was used here, was a modification of that of Knudsen. Fig. 1 is a longitudinal section of the type of vapor tube employed for selenium. It consists of a glass tube set into a larger one by a ground joint. The whole apparatus may be called simply the "tube," made up of an "inside tube" and an "outside tube." The latter has near the top an arm through which the air is pumped out. A lower chamber is separated from the remaining space in this tube by a thin perforated partition of platinum. The substance being studied is placed below the partition and kept at a constant temperature in a suitable oven. The inner, or "condensation," tube is fitted with a stopper through which pass 2 narrower glass tubes, one acting as a nozzle for the inflow of a cooling fluid, such as water, the other for its outflow. The vapor molecules pass up through the partition to the inside tube, where they are condensed as a solid deposit.

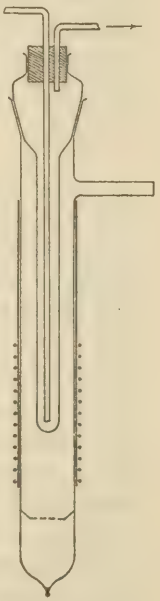

Fig. 1.

The vapor tube was inserted into a Freas oven through a hole drilled in the top, so that the tube projected into the oven space and was supported in place by its arm. The drilled hole was lined by a brass tube fitting it closely and providing a thermally conducting envelope. The platinum partition in the vapor tube extended one cm. or more below the roof of the oven chamber. Around the brass tube over this distance, the wire of an auxiliary heater was wound, to prevent condensation on the outside tube by maintaining a fairly uniform temperature up to the zone of deposit. Through the brass tube next to the ceiling of the oven chamber holes were drilled for a chimney effect, to give ready access of oven air to the vapor tube near the place of condensation.

A platinum resistance thermometer was hung in the oven chamber through a tube from above, so that the exposed helix of resistance wire was within about one cm. of the lower or vaporization chamber of the vapor tube when the latter was in place. A mercury thermometer also was passed through the wall of the oven chamber near the top, and its entire thread was always inside the oven. The bulb was within 2 cm.

[1] Brown, *Phys. Rev.*, 5, 236 (1915). See also Brown and Sieg, *ibid.*, 28, 497 (1914).

of the glass wall of the vaporization chamber. This thermometer was read through the glass door of the oven, and an allowance of 0.5° was made for parallax at each reading. The 2 thermometers thus occupied very nearly the same relative positions. In the earlier work the mercury thermometer was used as a check on the resistance thermometer, which it was at first proposed to use for the temperature readings at one-minute intervals, but it was unexpectedly found that the mercury thermometer was more suited to the purpose, with the other as a check. Temperature readings were taken in this manner for the present data.

A small electric motor outside the oven operated a rotary fan, placed with shaft vertical near the floor of the oven chamber, and with a diameter about $^2/_3$ the width of the oven. Motor and fan were at first connected by a spiral spring belt, but later by a shaft with friction drive. Thorough stirring of the air insured freedom from serious temperature gradient.

Preliminary Standardization of Tubes.

A vapor tension tube such as described can be standardized with some substance whose vapor pressure is known, by applying Equation 3, or by calculation from measurements of the tube dimensions according to Equations 1 and 2.

Effect of Poor Vacuum on Tube Resistance.[1]—Comparison of Equations 1 and 2 with results by the first method showed, as expected, that a poor vacuum introduces serious error. The effect of an air pressure large enough to render inapplicable the Knudsen expressions is shown in Table I(*a*). The resistance with roughly constant air pressure in the tube increases rapidly with the temperature, due to the collisions of the mercury molecules with those of the air.

Validity of the Knudsen Equations with Good Vacuum Conditions.—Of the two methods for determining tube resistance the second is more accurate, as Egerton points out, unless with the first method the mean of quite a number of determinations should be taken. It is comparatively easy to make accurate measurements of the dimensions for calculation of W in the second method.

The first method, however, was used in a preliminary checking of Equations 1 and 2, with mercury as the standardizing material. Two tubes used earlier in the work were standardized in this wy. For Tube A the values of W given in Table I(*b*) were obtained with the Langmuir pump. The mean value for this tube was 40.4 absolute units. The diameter of the circular apertures in the platinum partition was very nearly 2 mm. This value of the area gives 40.0 units for W, leaving but 0.4 unit for W_2, for which 2 units would have been more nearly correct. This would require 38.4 units for W_1, or a diameter of 2.04 mm. for the apertures.

[1] Dodd, *Phys. Rev.*, 11, 242 (1918).

Comparison of the diameter values for Tubes D, E, F, indicates that the diameters in Tubes A and B were likewise above 2 mm.

TABLE I.—VERIFICATION OF KNUDSEN'S EQUATIONS.

(*a*) Tube B, 6 apertures. Poor vacuum.

Temp. ° C.	*W*. abs. units.	Remarks. Determinations averaged.
87	227	12
97	294	3
106	329	3
120	629	2
129	771	3

(Approximate calculated value of $W = 15$ units.)

(*b*) Tube A, 2 apertures. Good vacuum.

Run No.	Temp. ° C.	*W* abs. units.
1	80	41.3
2	80	44.4
3	80	37.7
4	80	37.4
5	90	42.0
6	90	41.1
7	90	40.5
8	96	45.7
9	94	37.1
10	96	44.8
11	70.5	42.0
12	70	37.0
13	70	41.6
14	70	36.2
15	70	38.8
16	76	38.5
		Mean, 40.4

(Approximate calculated value of $W = 40$ units.)

That with good vacuum the effect of temperature on tube resistance is negligible was thus confirmed, and further, the value agreed with the Knudsen equations.

Calculation of Tube Resistances by Knudsen's Equations.

Calculation of W_1.—For the aperture measurements an enlarged photograph was made of the platinum partitions, with the glass broken away sufficiently to expose them. A millimeter scale was included in the photograph, for determining magnification. Measurements of diameters were made with Vernier calipers. The total aperture area for each tube was equal to A in Equation 1. Careful measurements of the individual apertures should have been carried out before the platinum partitions were sealed into the glass tubes. This was not done, as the holes were designed to be exactly 2 mm. diameter. Unfortunately, the method used in cutting them permitted slight departures from this specification.

Calculation of W_2.—The value of W_2 in (2) was determined for each tube from interior measurements of the dimensions. For the integrations a tube was regarded as divided conveniently into 4 parts, each part contributing its share of W_2. Thus, $W_2 = W_a + W_b + W_c + W_d$. In calculating W_2 it was necessary to decided what point should be taken as the upper limit, since the selenium condensed not only on the hemispherical end of the condensation tube, but to some degree spread up the sides. This upper limit of the part of the tube regarded as offering resistance to molecular flow was then taken as "the place of the deposit." The upper limit was in all cases put at one cm. from apex of condensation tube. This was done after a careful observation of the distribution of the condensed material, especially after the tube had been in continuous operation for an unusual length of time so that the thickness of the deposit was easily measured at various points with micrometer microscope, subsequent to breaking the deposit shell partially away from the glass.

Experimental Procedure.

The procedure in taking a measurement of vapor pressure was first to place the tube in the oven, at the same time starting the flow of tap water through the condensation tube. The pump system was not connected with the vapor tube until it had reached equilibrium temperature. At least 15 minutes, and usually more, was allowed for this, undoubtedly more than was necessary. The oil pump was then started and the pressure went down in 2 or 3 minutes to where the Gaede pump could be started. When this pump ceased to throw air up through the mercury at the revolving drum the time count by ordinary watch was begun. The apparatus was then allowed to run for 30 minutes, usually longer. Temperatures were read, as stated, at one-minute interval, and to 0.5°, and time was again read when air was let into the tube by disconnecting the short length of thick-walled rubber tubing between vapor tube and glass tubing connecting with Langmuir pump. The vapor tube was then removed from the oven, the inside tube removed from its place, and the selenium deposit scraped off on a previously weighed filter paper and weighed.

Data and Curves.

Data have been obtained for the vaporization curve of liquid selenium and the sublimation curve for the hexagonal crystals. Equation 3 was applied in the form

$$p = W.1/\sqrt{p_1}.G/t, \qquad (5)$$

where p_1 in Equation 3 has been neglected, since the vapor pressure of selenium at the condensation temperature must have been negligible compared with that near the melting point, some 200° higher. The condensation temperature was fairly constant at about 17°, and was that

of the tap water used for cooling. For the temperature corresponding to a given pressure measurement, and determining the value of p_2, the mean of the one-minute readings was taken. This average temperature was regarded as equal to a constant temperature that would have given the same amount of condensed material. The mean value was subjected to 2 corrections, one for thermometer calibration, the other for difference in temperature inside and outside the vaporization chamber of the tube. Both the mercury and the resistance thermometers had been standardized at the boiling point of C. P. naphthalene (Bureau of Standards sample) and the freezing point of water. Calibration was carried out under the same conditions as prevailed when the thermometers were in use in the oven. Both corrections of the mean temperature for a pressure measurement were conveniently made from a total correction curve. Since each of the 2 corrections is represented by a linear relation the total correction is similarly represented.

The factor ρ_1 in Equation 5 was evaluated graphically. The constant in Equation 4, $(\rho_1)_0$, was found for selenium by putting $n = 2$, and the vapor pressure curves as here presented were plotted on this basis (see later discussion).

The data as thus treated are assembled in Table II, for the "F series" of the selenium crystals and the "E series" of the liquid, together with data for some check measurements. The 2 letters, E and F, indicate the vapor tube concerned in each case. The F series consisted of 11 successive measurements, or "runs," under best conditions, and the E series, with the exception of 2 runs omitted due to experimental conditions going wrong, of 30 successive measurements.

A plotting of the p and T values as thus computed is inadequate for locating the melting point with any precision, even if it indicated the presence of a change in slope in the general curve, for this change, as is well known, is slight. But there is found from the present data a decided difference in the characteristics of the 2 curves when log p is plotted with $1/T$. This form is suggested for short temperature ranges by the first latent heat equation, as shown later, if L is regarded as constant.

Egerton treated his data in this manner. His temperature range for zinc was 106° and for cadmium 134°. The lowest temperature in the first instance was 162° below the melting point, while in the second it was 182° below. His plotted results showed some departure from a straight line, as might be expected. Assuming Kirchhoff's equation, after a preliminary assumption of a value for C in Equation 11, he could find the constants A and B by applying the relation at 2 experimental temperatures. In Egerton's work there was a gap of 56° between the highest temperature and the melting point for zinc, and of 48° for cadmium, and the relation for p and T had to be assumed over this gap to solve

TABLE II.—VAPOR PRESSURE DATA.

(*a*) F series, hexagonal crystals of selenium.

Tube F, $W = 12.41$; thermometer J.

Run No.	Aver. temp., ° C.	Corr. temp., ° C.	T, ° abs.	Total time, secs.	G, g.	G/t, $\times 10^4$.	$1/\sqrt{p_1}$ $\times 10^{-4}$.	p bars.	$1/T$ $\times 10^3$.	log. p.
1...	205.3	193.6	466.6	3000	0.0300	0.1000	1.566	1.94	2.143	0.2378
2...	209.8	197.6	470.6	2760	0.0422	0.1525	1.573	2.98	2.124	0.4742
3...	212.6	200.3	473.3	2205	0.0396	0.1795	1.577	3.51	2.112	0.5453
4...	215.9	203.2	476.2	2370	0.0506	0.2130	1.582	4.18	2.099	0.6212
5...	217.1	204.4	477.4	2610	0.0666	0.2550	1.584	5.01	2.094	0.6998
6...	219.4	206.5	479.5	2445	0.0690	0.2825	1.588	5.57	2.085	0.7458
7...	222.0	208.9	481.9	2130	0.0690	0.3240	1.592	6.40	2.075	0.8062
8...	223.9	210.6	483.6	2025	0.0800	0.3950	1.594	7.81	2.067	0.8926
9...	225.9	212.5	485.5	2010	0.0956	0.4650	1.598	9.22	2.059	0.9647
10...	227.5	214.0	487.0	1830	0.0914	0.5000	1.600	9.92	2.053	0.9965
11...	229.1	215.5	488.5	2070	0.1140	0.5500	1.620	11.05	2.047	1.0433

(*b*) E series, liquid selenium.

Tube E, 6 apertures, $W = 11.46$, thermometer J.

No. Run.	Aver. of temp ° C.	Corr. temp. ° C.	T ° abs.	Total time, secs.	G. g.	G/t $\times 10^4$.	$1/\sqrt{p_1}$ $\times 10^{-4}$.	p bars.	$1/T$ $\times 10^3$.	log. p
1...	231.7	221.0	494.0	2055	0.1360	0.6618	1.611	12.22	2.024	1.0870
2...	229.8	219.0	492.0	2950	0.2086	0.7071	1.680	13.61	2.032	1.1338
3...	238.6	228.0	501.0	2520	0.2184	0.8666	1.623	16.12	1.996	1.2074
4...	237.4	226.8	499.8	2400	0.2430	1.0125	1.621	18.80	2.001	1.2741
5...	244.1	233.7	506.7	2430	0.3408	1.4024	1.632	26.22	1.973	1.4186
6...	248.3	238.0	511.0	2355	0.3984	1.6917	1.639	31.77	1.957	1.5020
7...	242.1	231.7	504.7	2210	0.2826	1.2787	1.629	23.87	1.981	1.3778
8...	241.7	231.2	504.2	2520	0.3000	1.1904	1.628	22.20	1.983	1.3463
9...	242.5	232.0	505.0	2400	0.3034	1.2640	1.630	23.61	1.980	1.3731
10...	240.1	229.6	502.6	2400	0.3000	1.2500	1.626	23.29	1.989	1.3672
11...	233.3	222.7	495.7	2355	0.2046	0.8687	1.614	16.06	2.017	1.2057
12...	232.9	222.2	495.2	2470	0.2130	0.8623	1.613	15.935	2.019	1.2022
13...	235.4	224.8	497.8	2400	0.2430	1.0125	1.618	18.68	2.009	1.2714
14...	234.7	224.0	497.0	2460	0.2360	0.9593	1.616	17.77	2.012	1.2497
15...	234.0	223.4	496.4	2400	0.2240	0.9333	1.615	17.27	2.014	1.2373
16...	238.3	227.7	500.7	2400	0.2536	1.0566	1.622	19.64	1.997	1.2931
17...	238.0	227.5	500.5	2370	0.2556	1.0784	1.622	20.05	1.998	1.3021
18...	236.0	225.5	498.5	2400	0.2424	1.0100	1.619	18.73	2.006	1.2725
19...	236.4	225.8	498.8	1930	0.1926	0.9979	1.619	18.51	2.005	1.2674
20...	235.4	224.8	497.8	2910	0.2936	1.0089	1.618	18.70	2.009	1.2718
21...	234.5	223.8	496.8	2430	0.2534	1.0427	1.616	19.31	2.013	1.2858
22...	237.2	226.6	499.6	2460	0.2866	1.1650	1.621	21.63	2.001	1.3350
23...	234.8	224.1	497.1	1830	0.1642	0.8972	1.617	16.62	2.012	1.2206
24...	233.9	223.2	496.2	2385	0.2116	0.8872	1.615	16.42	2.015	1.2154
25...	232.0	221.3	494.3	2400	0.2078	0.8658	1.612	15.99	2.023	1.2038
26...	232.6	222.0	495.0	2400	0.1952	0.8133	1.613	15.03	2.020	1.1769
27...	235.1	224.5	497.5	2430	0.2286	0.9407	1.617	17.43	2.010	1.2413
28...	234.5	223.9	496.9	2400	0.2160	0.9000	1.616	16.67	2.012	1.2219
29...	236.7	226.2	499.2	2430	0.2418	1.0197	1.620	18.92	2.003	1.2769
30...	238.7	228.2	501.1	2400	0.2964	1.2350	1.623	22.97	1.995	1.3611

(*c*) Check Measurements.

Tube	No. of apertures.	*W*	Run No.	Aver. temp. ° C.	Corr. temp. ° C.	Total time, secs.	*G* g.	*G/t* × 10^4.	$1/\sqrt{p_1}$ × 10^{-4}.	*p* bars.
E	7	11.46	Ea1	219.7	208.6	3600	0.1050	0.2916	1.591	5.31
D	7	10.87	Da1	228.1	214.0	2415	0.0966	0.4000	1.600	6.95
D	7	10.87	Da2	218.6	204.6	2190	0.0360	0.1643	1.585	2.83
C	2	36.74	C1	243.2	228.7	2600	0.0988	0.380	1.624	22.67
C	2	36.74	C2	243.0	228.5	2460	0.0820	0.333	1.624	19.87

for the pressure at the melting point. In the present work the range for the solid was roughly 25°, and for the liquid 15°, while the pressure measurements were carried to within 2 or 3° of the melting point in both parts of the experiment, so that dependence need not be placed to any appreciable extent on an extrapolated curve for computations at the melting point.

The log p and $1/T$ relations from the present data for selenium and for a diatomic vapor are at least approximately linear, and there is a decided difference in slope between the two curves. To the extent that the relations can be regarded as linear and constants found on this basis, the melting point can be located with precision. But since the data have been obtained so near the melting point, considerable precision in locating it is to be expected anyway, as the extrapolation is not extensive. Even if the results thus plotted are segments of curves that are not linear, their slopes, readily found graphically, cannot differ appreciably from the slopes of the true curves over the experimental temperature ranges, that is, in a limited region including the melting point.

TABLE III.

n.	*f*.	$(p)_n$ m. p. bars.	L_1.	L_2.	L_3.	$(dp/dT)_1$	$(dp/dT)_3$
			calories per gram.			bars per degree.	
2	...	12.68	135.5	83.9	219.4	0.57	1.05
4	0.707	8.95	67.7	42.0	109.7	0.40	0.74
6	0.577	7.31	45.1	28.0	73.1	0.33	0.60
8	0.500	6.34	33.9	20.9	54.8	0.28	0.52
10	0.447	5.66	27.1	16.8	43.9	0.25	0.46
12	0.408	5.17	22.6	14.0	36.6	0.23	0.42

Data for the log p and $1/T$ relations are given in Table III. For the F series the location of the mean straight line was quite easy. Even for the E series for the liquid, where the plotted points are more erratic than in the F series, there can be little doubt in locating the mean straight line when this is based on the points with lower values of mean temperature deviation during a run, or those where the deviation was less than 0.3%. These points were 16 in number, *viz.*, 5, 6, 8, 11, 13, 15–19, 22, 24–28. While temperature variation was not the only source of error more confidence was placed in those runs where this variation was not large. At any rate the points for the remaining runs group themselves well about the mean straight line as thus drawn, particularly points 4, 7,

9, 14. The points lying farthest removed from the line above it, with the single exception of 22, had larger temperature variation, *viz.*, 10, 13, 20, 29, 30. The points corresponding to the first 3 runs of the series may be neglected, as the required vacuum was slowly reached in all 3, due probably to gases coming off from the molten selenium. In the 4th run, however, the vacuum was not readable on the McLeod gage after 7 minutes. Vacuum conditions for the 6 runs of larger temperature variation were good, with the possible exception of 20, but even in this case they were obviously not poor enough to interfere with molecular flow. The mean straight line as determined from the 16 points of lower temperature deviation was, therefore, accepted.

The equations for the 2 lines are,

$$\text{F series: } \log p = -7644 \times 1/T + 16.692 \qquad (6)$$

$$\text{E series: } \log p = -4722 \times 1/T + 10.733 \qquad (7)$$

The values for y and x for the point of intersection give, respectively, $p_{\text{m. p.}}$ in abs. units, and the melting point itself on the absolute scale. As thus found, $p_{\text{m. p.}} = 12.7$ bars (0.00957 mm.), and melting point = 217.4°.

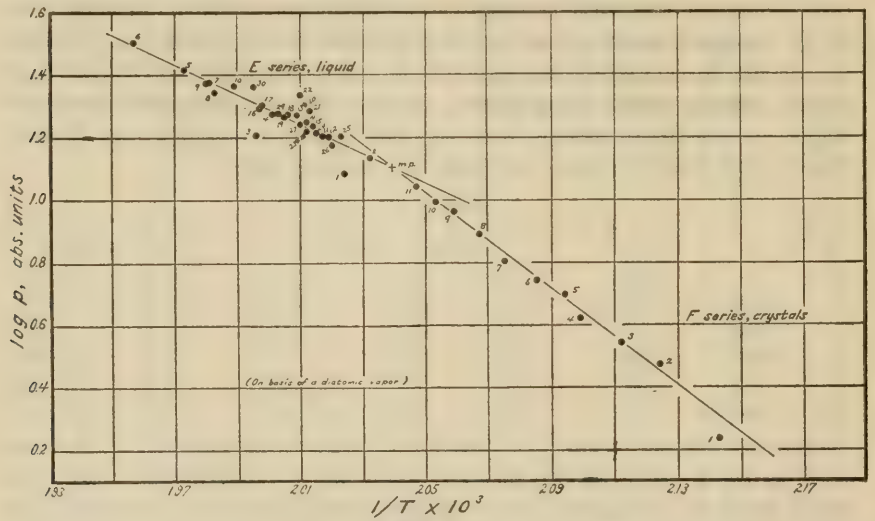

Fig. 2.

The exponential relations corresponding to Equations 6 and 7 are of the form

$$p = 10^{(b+m/T)} = c_1\, 10^{c_2/T} \qquad (8)$$

where m is the slope and b the y-intercept of the log p and $1/T$ line whose equation is

$$\log p = m.1/T + b \qquad (9)$$

Equations 8 and 9, with the numerical values above, are empirical ex-

pressions for the temperatures of the present experiments, and on the basis of a diatomic vapor.

From the slopes of these 2 lines, Fig. 2, the slopes of the p and T curves may be found by

$$dp/dT = -2.3\, m/T^2 \times 10^{(b+m/T)} \text{ bars/degree} \qquad (10)$$

The values at the melting point were,

for the solid, $dp/dT = 1.050$ bars/degree

for the liquid, $dp/dT = 0.573$ bar/degree

The two p and T curves assembled in Fig. 3 were plotted from Equation 8. The triangles indicate points computed from the empirical expressions.

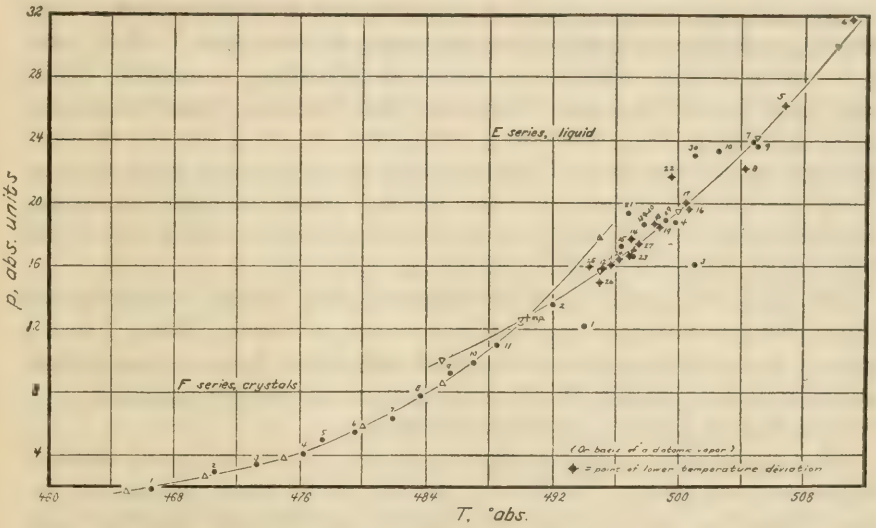

Fig. 3.

Five check values of pressure, with different tubes, are given with data in Table II(c). The 3 check values for the solid curve are all low for reasons known. Check measurements C_1 and C_2 were of most value, for they were obtained with a tube having but 2 apertures as contrasted with Tube E with 7 apertures. It was desired to make more check measurements with this tube at different temperatures, but unfortunately an accident during a third run destroyed the platinum partition. But the 2 measurements C_1 and C_2 are regarded as satisfactory. In addition there was the check on the method in the preliminary work with mercury already described.

Sources of Error.

There are various sources of error in the molecular flow method as here used, some of them of sufficient importance to be particularly guarded against.

To prevent serious temperature gradient in the oven a fan was operated as described. The temperature during a run was as constant as the thermostat of the oven gave. The average deviation from the mean for all 41 measurements of both series was 0.357%. The pressure value is affected by temperature variations in 2 ways, directly through ρ_1, and indirectly by giving an experimental value of G that does not correspond to the mean tube temperature and is likely to be too large, for the pressure of a saturated vapor rises more rapidly than the temperature. Error through ρ_1 can be neglected, since this factor is but slowly variable with temperature. As to the error through G, the indications are that it is small.

The important temperature correction for difference inside and outside the tube at the vaporization chamber was made for Tubes D, E, F, with C. P. tin and silver nitrate crystals as standardizing materials. These were used inside evacuated thin-walled glass Tubes as usual for melting point determinations. They were suspended one at a time in the central part of the vaporization chamber with the vapor tube in the oven as for a pressure measurement. The melting points of these substances were taken at 232 and 209°. In future work a thermocouple sealed into the vapor tube might be found useful and preferable.

The method of molecular flow requires a good vacuum, otherwise the tube resistance cannot be regarded as constant, as already shown. In the present work the vacuum soon reached a desirable degree, 0.00005 mm. or better, as indicated by the McLeod gage, which became useless for precise reading of the lower pressures obtained.

Another difficulty in this method is the uncertainty regarding when to begin the time count. Ideally the vacuum would reach the desired degree at the time zero, and the error will be least when this time is a minimum. Experience has shown that the Langmuir pump reduces the pressure in the vapor tube to a value permitting molecular flow in a brief interval, varying somewhat, possibly because of various amounts of adsorbed gases coming from the apparatus itself and so depending to some degree on the time of standing of the tube between runs. But, as a general rule, after 6 minutes, when the McLeod gage was usually read for the first time in a run, the pressure was less than 0.0002 mm., much below that necessary for molecular flow. In several cases noted, particularly E4, the vacuum was not readable after 7 minutes. But a study of the results shows no consistent variation with time of run.

For most precise measurement of aperture areas they should be perfectly circular, which was not completely realized. Since W_2 was in no case greater than 16% of W_1 any departure of the tube from the cylindrical has been neglected. Integration, however, was carried out for change in shape of cross-section. The distance the deposit spreads up

the sides of the tube increases somewhat with temperature, but this slight increase was neglected, as was likewise any error in W_2 due to error in dimensional measurements. There may be some error in the resistance values due to locating the place of condensation in the manner already described, but no attempt has been made to correct for it. A test of the chemical balances showed an error of 0.3%, also neglected.

Adsorbed moisture gathered on the deposit between time of removal from condensation tube and time of weighing. Evidence of this was obtained on a humid day. For E3 the weight of the deposit had increased 0.91% in 1 hour, and 2.1% in 2 hours following the first weighing; E4 had increased 1.56% in 2 hours; E5, 0.41% in 2 hours; while E6, permitted to stand all night for a test weighing, showed no change in 11 hours. Evidently the humidity fell as the day advanced. E10, a run made 2 days afterward, showed an increase of 0.47% in over 2 hours following the first weighing, and E13, made later in the same day, showed a 0.41% increase in 11 hours. Usually the time elapsing from removal of tube from oven until the weighing, amounted to from 10 to 40 minutes. Measurements were not taken to determine this disturbance in all cases. It was noticeable that even with E3, where the adsorption rate appeared to be a maximum, the plotted point on the p and T diagram was low, due to other causes which more than offset error due to adsorption.

It was assumed that the outside tube was kept at the same temperature, at least up to a point opposite the place of deposit on the condensation tube. The precautions taken have been described.

The Kirchhoff Equation.

The Kirchhoff equation has the form

$$\log p = A - C \log T - B/T. \qquad (11)$$

It may be derived from the Clapeyron-Clausius equation by integration after assuming a linear relation. By assuming certain constants for this relation there is obtained

$$\log p = k/2.3 - L_o/2.3R_1.1/T - (c - c_p)/R_1.\log T \qquad (12)$$

proposed by Hertz for mercury.

Knudsen found that this form applied to empirical results at higher temperatures, when extrapolated gave pressure values in good agreement with those obtained by him at lower temperatures by the molecular flow method. He used for mercury the value 0.847 for C in Equation 11, from experimental values of the specific heats. Egerton assumed for both zinc and cadmium the simpler value of 0.5 for this constant, based on theories of Boltzmann,[1] where both solid and vapor are monatomic.

[1] Partington, "Thermodynamics," 1914, p. 17.

While admitting that this value cannot be true Egerton[1] justified it on the ground that it would make possible the simplest approximation for a comparison of results with different elements. But the specific heats of selenium have not been determined. Moreover, the vapor is probably not monatomic.

It is not necessary to regard the variability of L for short temperature ranges as in the present work. These ranges are not sufficient here to determine how strictly the Kirchhoff type applies to selenium. The present data do not conflict with that type.

It seems desirable to let the results stand for the present expressed by the simple exponential relation, which will suffice for a limited temperature range near the melting point, and to regard it as empirical, with the understanding that it is based on a diatomic vapor, although to the extent that it is exponential the number of atoms assumed per vapor molecule does not change the shape of the curves. That this simple relation may be taken to hold quite generally for temperatures considerably below the critical has been stated recently by Richards.[2]

The Three Latent Heats at the Melting Point.

From the empirical relation of Equation 10, $1/p.\mathrm{d}p/\mathrm{d}T.T^2 = -2.3m$, and from the latent heat equation (in the form $L=R/m.T^2/p.\mathrm{d}p/\mathrm{d}T$ the same quantity is equal to L/R_1 (see also Richards[3]). Equating

$$L = -2.3R_1m. \qquad (13)$$

Thus the latent heat is simply obtained from the slope of the log p and $1/T$ curve, by combining the theoretical latent heat equation with empirical results. The latent heat equation can then be used to find the latent heat at any temperature, provided the slope of the true log p and $1/T$ curve is known at that temperature. The actual relation between log p and $1/T$ can be regarded as linear over a limited segment of the true curve to the extent that the latent heat can be regarded as constant over the same limited temperature range. Obviously then, when Equation 13 is applied near the melting point the value of L is the value of L_o in Equation 12. Now if the Kirchhoff equation (11) holds, the log p and $1/T$ curve can vary from a straight line only through the log T term. It follows from Equation 13 that the physical meaning of the variability of this term would be the variability with temperature not only of the difference in the specific heats but also of the latent heat. Otherwise m in Equations 10 and 13 and B in 11 would be identical. As it is, $m = -L/2.3R_1$, while $B = -L_o/2.3R_1$.

[1] *Loc. cit.*, p. 43.
[2] Richards, *J. Franklin Inst.*, **187**, 584–5 (1919).
[3] *Loc. cit.*

Treatment of Data on Basis of Different Numbers of Atoms Per Molecule of Vapor.

At 700° or 800° selenium vapor is apparently diatomic,[1] but that of itself probably tells little of conditions near the melting point. In the absence of direct density measurement of the vapor at the experimental temperatures, the 3 latent heats, the pressures, and the slopes have been computed at the melting point for 6 cases, depending on the number of atoms per molecule. See Table III.

Let n be number of atoms per vapor molecule. This enters the factor ρ_1 which by Equation 5 affects the pressure, while W is a constant of the apparatus and G and t are both experimental. Although ρ_1 depends also on temperature that is likewise experimental. Let $(\rho_1)_2$ be density at unit pressure for the data as already computed for a diatomic vapor. From Equation 4 by Avogadro's law, $1/\sqrt{(\rho_1)_n} = \sqrt{2/n} \cdot 1/\sqrt{(\rho_1)_2}$, and from Equation 3, $p_n = \sqrt{2/n}.p = f_p$, where p is pressure for a diatomic vapor and f is the conversion factor. For the melting point and $p_{\text{m. p.}}$, in Equation 8 $c_1 = 10^b$ and $c_2 = m$. The log p and $1/T$ relation remains linear if the initial relation with $n = 2$ is taken as linear, and the slope c_2 in Equation 8 is unchanged in the conversion, while the y-intercept is varied by an increment, log f. On this basis then the melting point does not depend on n; but $p_{\text{m. p.}}$, and the latent heats as computed from the vapor pressure data, do depend upon it. L changes only through R_1, and from the values of L already computed, $L_1 = 271/n$, and $L_3 = 438.8/n$. Also, $dp/dT = K'p$, so that $[(dp/dT)_1]_{\text{m. p.}} = 0.9451\ p$, and $[(dp/dT)_3]_{\text{m. p.}} = 0.0828\ p$. Pending a determination of the vapor density this treatment will serve for a simple approximation. The computed pressure values in Table III may be regarded as depending upon experiment rather than theory, since the Knudsen equations have been experimentally confirmed.

Summary.

1. By the method of molecular flow data for the sublimation curve for selenium crystals of the hexagonal system have been obtained over the approximate temperature range from 190° to 215°.

2. By the same method data for the vaporization curve for the liquid of vitreous, amorphous selenium have been obtained over the approximate temperature range from 220° to 235°.

3. The relation between vapor pressure and temperature for Se over the temperature ranges studied and on the basis of a diatomic vapor may be expressed by a simple exponential relation, $p = c_1 e^{c_2/T}$, where T is absolute temperature.

4. On the basis of a diatomic vapor the results appear to locate the

[1] See Roscoe and Schorlemmer, "Chemistry," 1, 469 (1911).

melting point at 217.4°, and give a pressure value at the melting point of 12.68 bars (0.00954 mm.), and slope values for the p and T curves at the melting point of 1.050 bars/degree (0.000767 mm./degree) for the crystals, and 0.573 bar/degree (0.000431 mm./degree) for the liquid.

5. The slope at the melting point was thus found to be greater for the solid than for the liquid, as required by thermodynamics, and as found for other substances. The difference between the 2 slopes is seen to be 0.477 bar/degree (0.000358 mm./degree) on the basis of a diatomic vapor.

6. From the first latent heat equation (Clapeyron) of thermodynamics, the 3 latent heats at the melting point were computed on the basis of a diatomic vapor as follows, for vaporization, 135.5; for sublimation, 219.4; and for fusion, 83.9 cals./g. Since the density of the vapor at the present experimental temperatures has not been determined, the 3 latent heats, the pressure at the melting point, and the slopes of the p and T curves at the melting point, have been computed on the basis of 4, 6, 8, 10, and 12 atoms per vapor molecule. The computed latent heat values vary inversely as these numbers.

7. The results are not out of harmony with the requirements of the Kirchhoff equation, based on thermodynamics, relating vapor pressure and temperature, but they do not give positive evidence that this equation is applicable to selenium, as would be expected from the limited temperature ranges even if that relation should apply.

8. As a preliminary to the present work the Knudsen equations for resistance to molecular flow have been further verified, with mercury as the standardizing material.

In conclusion, the writer wishes to express his thanks to the members of the Physics Department of the State University of Iowa for their interest. Particularly is he indebted to Dr. F. C. Brown, who suggested the problem, and under whose direction the research has been carried out, and also Dr. G. W. Stewart, who has contributed materially to its completion. His gratitude is also due the instrument makers, M. H. Teeuwen, and his successor, at the University, J. B. Dempster.

Iowa City, Iowa.

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A NOTE ON THE CORRECTION OF CONTACT DIFFERENCE OF POTENTIAL DEVELOPED IN COMPTON'S MODIFICATION OF THE QUADRANT ELECTROMETER.

BY OTTO STUHLMAN, JR.

SYNOPSIS.

A method of eliminating contact difference of potential in Compton's modification of the quadrant electrometer is suggested. The quadrants must not be built up. Solder, fillers or finishers must be omitted. If the needle is of aluminium the quadrants must be of aluminium, milled out of a single piece of metal. The final curve shows the absence of contact difference of potential in such an instrument when reconstructed and possessing an aluminium needle and milled aluminium quadrants.

IN an article on "A Sensitive Modification of the Quadrant Electrometer"[1] it is said that "the most serious sources of difficulty appear to be irregularities in the needle, or quadrants, and contact difference of potential between the quadrants," and further that this effect is proportional to the first power of the needle potential. Incidentally it is also stated that "such contact differences of potential, if found troublesome, may be removed by cleaning (the quadrants) or compensated by a small potential permanently applied to the 'earthed' quadrants."

Unfortunately however, many conditions may arise when it is neither possible or desirable to have a compensating potential permanently applied to the "earthed" quadrants. In these circumstances the contact difference of potential existing in the quadrant system of the instrument must be removed by structural changes. The simplest and most obvious changes would be to make all the quadrants as well as the needle of the same metal. The quadrants should not be made of several pieces soldered together as in the "Compton Electrometer," but should be cut from the solid metal and then machined to the proper size and shape.

In Fig. 1, are shown three examples to indicate how, under electrostatic positive control, the scale deflections vary as the potential across the quadrants is increased. The scale deflections, proportional to θ, are plotted in Figs. 1, 2 and 3 so that, as in Fig. 1 for example, if the large deflections are to be strictly interpreted they must be read $\tan 2\theta = d/D$, where d is the deflection and D the distance of the scale

[1] A. H. Compton and K. T. Compton, PHYS. REV., Vol. 14, Aug., 1919.

from the mirror. For the large deflections this correction is about one per cent. The potentials were measured by means of a standardized Simens and Halske millivoltmeter. The needle of the instrument was of aluminium foil and the quadrants of brass. Each of the latter were m de of three pieces, top and bottom soldered to a circular vertical band.

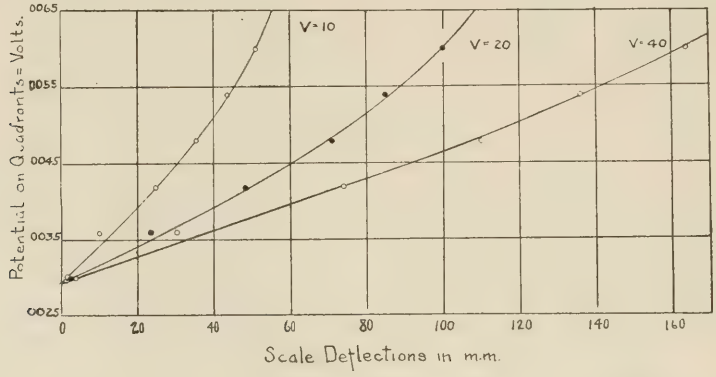

Fig. 1.

If now the needle is kept at a definite potential and the deflections observed as the potential on the quadrants are increased, it is found that the curve would not pass through the origin. Changes in the potential of the needle say from forty to twenty and finally to ten volts developed curves which possessed approximately equal intercepts on the voltage

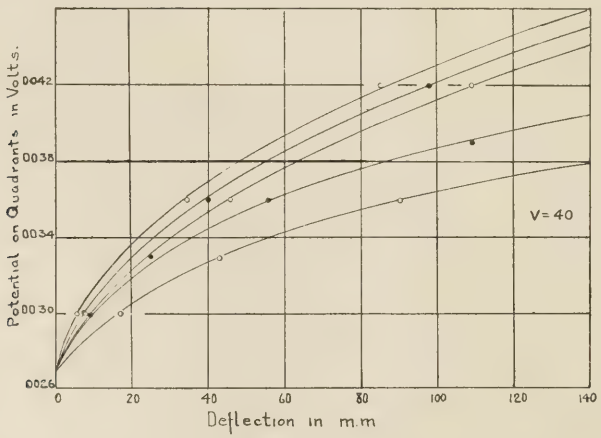

Fig. 2.

axis. This value was found to be 0.00292 volts. The quadrants were next adjusted to produce negative electrostatic control and the above observations repeated. The intercept of these curves had the same value as that of the above curve.

The quadrants were then removed, thoroughly cleaned in acid and scraped to remove all visible indications of solder. The observations were repeated and the voltage intercept was found to have dropped to 0.00264 volts. In Fig. 2 is shown a typical set of results obtained under these conditions. They represent the quadrants adjusted to negative electrostatic control with 40 volts on the needle. The successive curves show that the intercept is independent of the degree of negative control. The upper curves were obtained under less electrostatic control than each of the successive lower ones.

Hence variation in the degree of electrostatic control or potential on the needle does not change the value of the voltage intercept.

Further cleaning and scraping of the quadrants did not reduce this value below 0.00170 volts. A tilt of the needle about its minor axis may account for the fact that the curves do not pass through the origin.

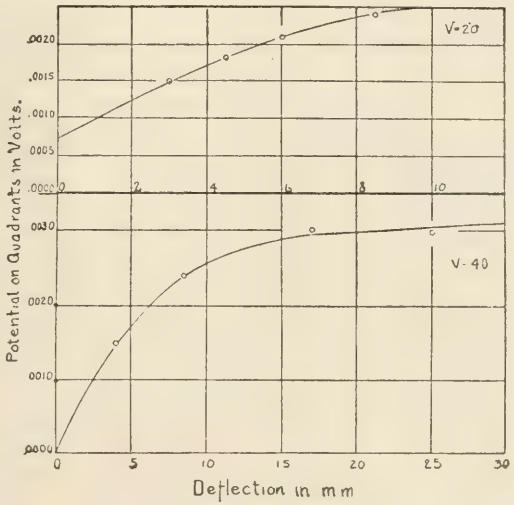

Fig. 3.

This explanation was eliminated because the same needle without variation in its tilt was used throughout the observations. A contact difference of potential in the system would account for it, especially in view of the successive changes in the value of the intercept, as the physical conditions of the inner surfaces of the quadrants were changed. In order to test this hypothesis, the quadrants were replaced by aluminium ones constructed as suggested above, the aluminium foil needle retaining its original tilt. This structural change produced the results shown in the upper curve of Fig. 3. The supposed contact difference of potential has been reduced to 0.00070 volts. An examination after twenty-

four hours and all later examinations showed no contact difference of potential and a typical curve developed under these conditions and under negative electrostatic control is shown in the lower half of Fig. 3. The curve passes through the origin. The subsequent drop from 0.00070 to zero volts is of course accounted for by the occlusion of air on the new surfaces of the aluminium quadrants, ageing[1] them to the same surface conditions as existed on the very much older aluminium foil needle.

A further point of interest in the above curves is the lack of uniformity in the sensitiveness for all values of the needle deflection. This has been pointed out by Compton, in that "the presence of the term $k_5 V^2\theta^2$ in equation (5) shows that the sensitiveness is not the same for all values of θ, *i.e.*, over all parts of the scale, though this term is not important, except at very high sensitiveness." Unfortunately no curves for this variation are shown in the above paper so that no comparison is possible. How close these curves approach to a straight line can be seen in the figures here presented. The best results were those obtained in Fig. 1, when the needle was charged to 40 volts and the quadrants adjusted to electrostatic positive, though nearly zero, control.

The data for this note was obtained while the writer was a member of the staff of the State University of Iowa.

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[1] See "On the Influence Contributing to the Variation of Contact Electromotive Force with Time," O. Stuhlman, PHYS. REV., Vol. 8, p. 294, 1916.

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THE PERFORMANCE OF CONICAL HORNS.

BY G. W. STEWART.

SYNOPSIS.

The experiments were performed to secure quantitative results with a conical horn used as a receiver. The approximate theory of A. G. Webster, when extended, and the experimental data herein presented lead to the following conclusions concerning such a horn acting as a receiver:

Intensity at Vertex as a Function of Frequency.—The experimental variations of intensity at the vertex plotted with changing frequency, intensity of the arriving waves constant, demonstrate that:

(*a*) The frequencies producing resonance are very closely integral multiples of the fundamental frequency.

(*b*) The resonance peaks broaden and the minima between the peaks increase with increasing frequency.

(*c*) Therefore the conical horn used as a receiver gives an amplification for any frequency nearly the same or greater than the fundamental.

(*d*) The greater the frequency, the less the differences between maxima and minima, and thus a horn long compared to the wave-length of the least frequency involved, gives an amplification without highly marked resonance characteristics.

(*e*) The function of frequency obtained experimentally varies with the horn angle chosen, but retains the same general character.

Optimum Horn Angle for Fundamental Resonance.—There is a horn angle for which the amplification will be a maximum, assuming fundamental resonance. The optimum angle is greater the higher the frequency, 256 and 512 d.v. being used.

Optimum Horn Angle; Overtones.—Using 512 d.v. as the fundamental and first and second overtone, similarly 256 d.v. as the fundamental and first overtone, the optimum horn angle decreases with the order of the harmonic, being greatest for the fundamental.

End Correction.—The end correction of the conical horn is approximately 0.7 of the radius.

Conical Attachments.—Cylindrical and conical attachments to a conical horn introduce their own resonance characteristics, which may be estimated as to frequency by treating the end attached as a free open end but applying a positive correction to the length if the angle of the attached tube is less, and a negative correction if greater.

Limitations of Conclusions.—The conclusions apply strictly to the limits of frequency used in the experiment. Nevertheless they make more clear the nature of the action for all frequencies.

THE published theory[1] of the action of conical horns and pipes is unsatisfactory in at least two respects, namely, there is a masking of certain physical aspects of the action through the assumption of spherical waves, and there exists too great an inaccuracy in these com-

[1] See Rayleigh's Theory of Sound, Vol. II., Ch. XIV., and Barton, Phil. Mag., Series 5, Vol. 15, 1908, pp. 69–81, and Webster, Proc. Nat'l. Acad. of Sciences, Vol. 5, p. 275, 1919.

puted quantitative relations which should enable one to antitipate the amplification produced by a conical horn when used as a receiver. The present contribution furnishes certain quantitative information concerning the action of conical horns as receivers, under conditions as specified, and also gives a brief explanatory discussion expressed in terms of the physical action involved.

I. Apparatus and Methods.

Sources of Sound.—The sources of sound were two electrically driven tuning forks mounted upon wooden box resonators having openings of 10 x 5 cm. for the 256 fork and 7.3 x 3.7 cm. for the 512 fork.

General Method of Measurement.—A Rayleigh disc, connected by rubber tubing to the conical horn investigated, was used to obtain the relative intensities of the sound in the conical horn. The internal diameter of the rubber tubing was .45 cm. and the length about 150 cm. Readings were taken by the means of a telescope and scale. The deflections of the disc were proportional to the intensities in the disc and hence to the intensities at the vertices of the horns. A disc was constructed for each frequency, the upper one shown in Fig. 1 being for the 512 fork.

Location.—The source of sound, horn and disc were mounted on the edge of the roof of the physical laboratory 21 meters high, where the

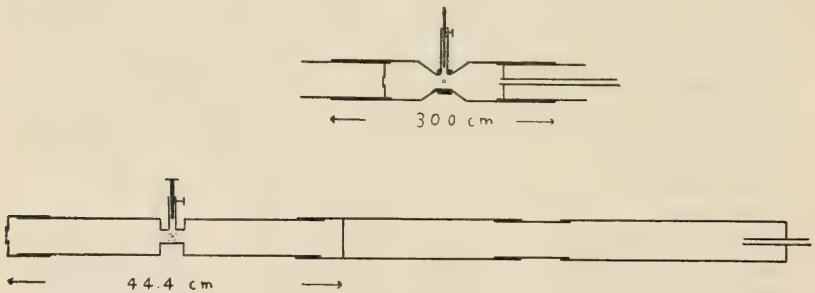

Fig. 1.

absence of other buildings, combined with the elevation of the site, made the reflection of sound nil, excepting that from the root itself. This reflection was diminished in two ways: first, by covering the adjacent portions of the roof with thick hair felt whose absorption coefficient is about 50 per cent. and by mounting the fork and horn approximately 150 cm. above the roof and separating them from each other not further than this same distance. Thus the effect of roof reflection was reduced to a small per cent. However, the proximity of the horn and fork

introduced an error in that the wave falling upon the horn was not strictly plane, though in all cases the wave surface deviated from a plane only by a small fraction of a wave-length.

Horns.—The horns were made of galvanized iron, U. S. standard plate No. 24. It was found that, if thin metal were used, there was resonance in the conical wall affording sufficient dissipation to affect the intensity in the horn itself. This effect could be demonstrated by changing the rigidity of the clamping of the horn. With the horns actually used, however, the intensity measurement was not capable of being changed by clamping and the walls did not vibrate noticeably. Inasmuch as the absorption of the metal when not vibrating is very small, it is assumed that the results here presented are those found in conical horns having rigid walls and possessing no absorption.

Each horn was cut off at the vertex, leaving a diameter of 0.5 cm.; thick wall rubber tubing 0.45 cm. in internal diameter and approximately 150 cm. in length, was slipped over the open vertex and connected to the Rayleigh disc.

In many of the experiments the lengths of the horns were altered a few centimeters in order to produce the length required for resonance. This was accomplished by means of heavy drawing paper slipped over the large end. It was experimentally shown that when the length of the addition of drawing paper did not exceed a few centimeters the paper served just as well as metal itself. The reason is, of course, that the changes in pressure near the large end of the horn are small.

Detailed Method.—Deflections of the disc were read and assumed (and justifiably so) to be proportional to the intensities in the horns. But in order that measurements of relative intensities with varying experimental conditions might be secured, corrections were made for the lack of constancy in the intensity of the source of sound, and for the change of sensitiveness of the disc caused by changes of temperature. The data for the first correction were obtained by microscopic measurement of the amplitude of the fork at the time of each reading of the disc. The second correction was secured through an occasional reading of the disc when attached to a tube terminating in a position fixed relative to the source of sound, the amplitude of the fork being noted. The details of applying these corrections are simple and almost obvious. The only assumption involved is that the intensity is proportional to the square of the amplitude of the fork.

II. EXPERIMENTAL RESULTS.

The experimental results herein presented are not as accurate as can be obtained, but they will give fairly satisfactory quantitative information concerning the action of conical horns. The experimental conditions

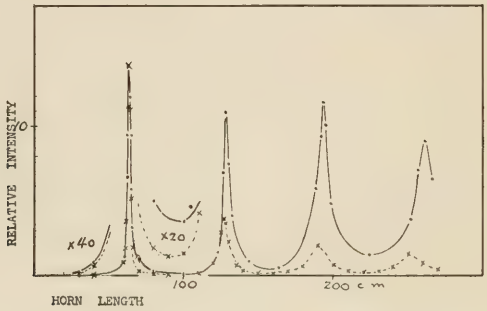

Fig. 2.

necessary to obtain more accurate results are difficult to secure. The experimental points shown in the curves are usually each the result of a single setting and observation. Nevertheless, they do not comprise the entire data but are merely typical. All results have been corrected for temperature, the reduction being made to 20° C.

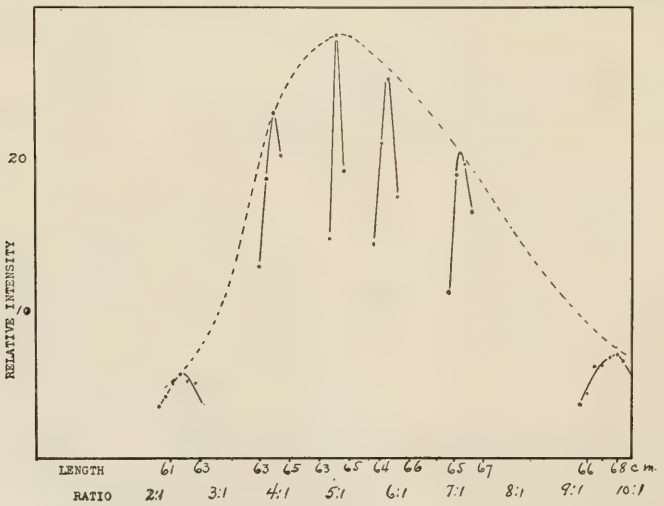

Fig. 3.

Variation of Intensity with Length, Frequency Constant.—The first question one might ask in regard to the action of a conical horn or tube acting as a receiver is in regard to the resonance characteristics and the

amplifying effect throughout the range of frequencies of interest. A series of trials was made with conical horns of constant angle but of varying length, using the same source of sound. The data were taken on several evenings and are shown, after corrections are made for the

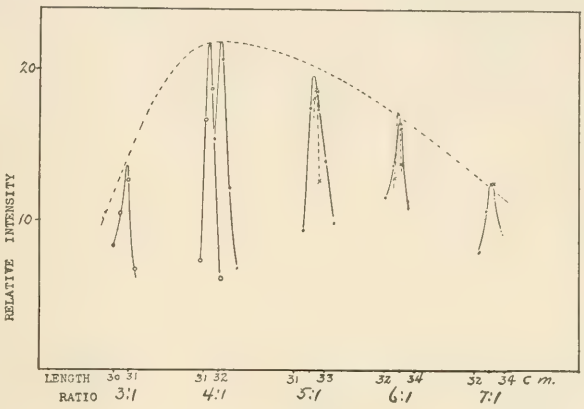

Fig. 4.

effect of temperature differences upon length, in the full line curve plotted in Fig. 2. Over two portions of the curve magnifications of 40 and 20 are also plotted. There should be added to the data in Fig. 2 an important relevant fact, namely, that the intensity without the horn as tested by an open tube at the position occupied by the open end of the

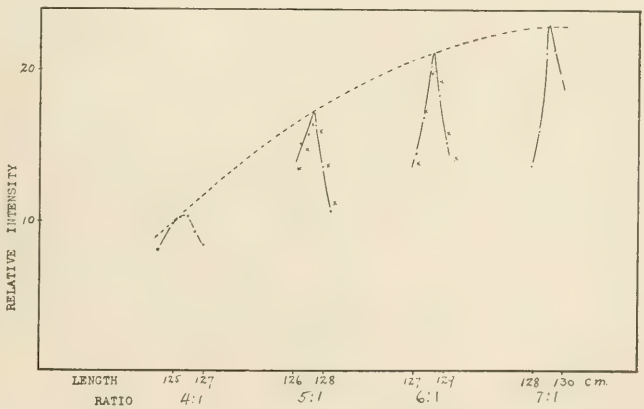

Fig. 5.

horn, was of the same order as the observation with the horn of shortest length.

Variation of Intensity with Angle in a Conical Horn, Assuming Maximum Resonance and Fundamental Frequency.—In Fig. 3 it is shown that

the maximum intensity at resonance occurs when the ratio of radial length to diameter of opening is approximately 5 : 1 for a frequency of 256 d.v.

Fig. 4 gives similar information in regard to the frequency of 512 d.v. Here the optimum ratio is approximately 4 : 1, that is, the radial length if four times the diameter of the open end. The observations indicated

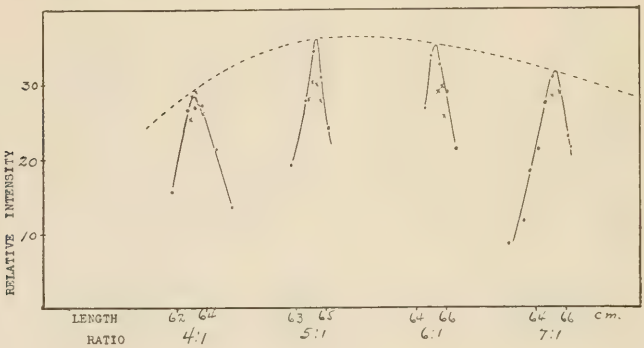

Fig. 6.

by circles and crosses were taken at other times. There appears a disagreement of peak location for the 4 : 1 probably due to an error in reading of 1 cm.

Resonance Peaks, Similar to those in Figs. 3 and 4, but with a Frequency of the First Overtone.—Fig. 5 for 256 d.v. and Fig. 6 for 512 d.v. are self-explanatory. Again crosses and dots indicate separate series. The point worthy of note is that, frequency constant, the optimum angle decreases when the horn is lengthened so that the fundamental becomes the first overtone.

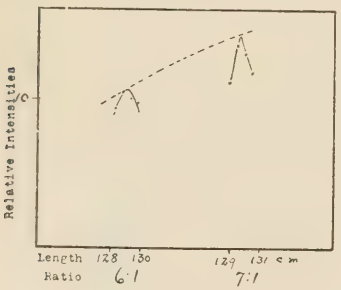

Fig. 7.

Resonance Peaks, with Frequency that of the Second Overtone.—With 512 d.v. only observations on two horns were taken and these are shown in Fig. 7. Although these data are too meager to interpret accurately, yet there is sufficient evidence that the optimum angle continues to decrease with an increase in the order of the overtone.

Variation in Optimum Angle with Order of Overtone.—Fig. 8 shows how the optimum angle changes with the order of the overtone and it is here seen that the ratio of change for the 256 and 512 is approximately the same. The reliability of the data gives only a general idea of the change.

Effect upon Resonance Characteristics of Conical and Cylindrical Attachments.—It is to be anticipated that the resonance characteristics of the tube attached to a conical horn for listening purposes will be superimposed upon the characteristics of the cone itself. By simple tests with a tonometer and attached cones and cylindrical tubes the writer found that the resonance frequencies of the attachments can be estimated by the following rule: The attached tube, cylindrical or conical, will introduce resonance characteristics which may be computed as to

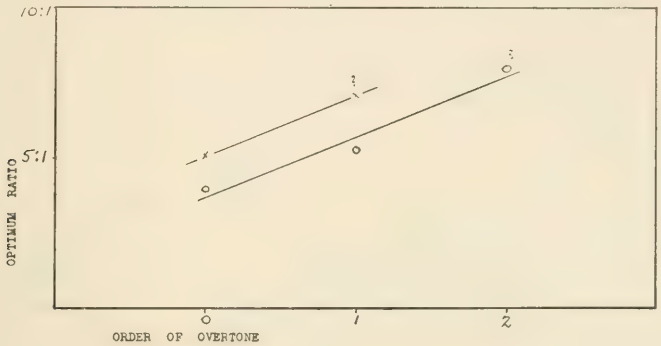

Fig. 8.

frequency by treating the end attached as a free open end but at the same time remembering that if the angle of the attached tube is less than that of the conical horn there is a positive end correction and if greater a negative end correction.

Directivity of a Conical Horn.—As is well known, when the source of sound is along the extended axis of the horn, the greatest intensity is produced at the vertex. This effect, because it depends upon the phase relations at the opening, becomes more marked the larger the opening of the horn. This comment is introduced merely because in practical application of the conical horn as a receiver, the exclusion of side noises is of great importance.

End Correction.—The experiments may be used to determine, the end correction of a conical horn. If we assume that,

$$L + KR = n\frac{\lambda}{2},$$

where L is the length of the horn, R the radius, K a correction factor, and n an integer, and plot L/n, and R/n as coördinates, K may be computed from the slope. If the results are in agreement with the above linear relation, it may reasonably be concluded that the resonance frequencies are integral multiples of the fundamental. The results of the experi-

ments have been so plotted and the assumption of the linear relation seems to be fully justified, whether the frequency is a fundamental or an overtone. The value of K appears to be 0.7, with a possible error of 10 per cent. Experiments can readily be designed to determine the exact relation between the resonance frequencies of the conical horn, and thus the end correction, with a much greater accuracy than here employed.

III. THEORY.

A brief reference to theory will aid in extending the application of the experimental results and in understanding the action of the conical horn. The only theory of which the writer is aware that offers a possibility of obtaining quantitative results for conical horns is that of Dr. A. G. Webster[1] of Clark University. In this theory he makes the following assumptions:

(*a*) That the tube or horn has a cross-section which is always small but is a function of the length, x.

(*b*) That the sound waves travelling in positive and negative directions of x are plane, *i.e.*, all variations in y and z directions are zero.

(*c*) That the conductivity for an open end is the same as a channel of a selected length.

(*d*) That the general equations of sound are applicable.

It will be necessary to extend the published work of Webster somewhat in order that we may secure the quantitative results. Let X be the volume of air periodically entering the horn under a maximum excess pressure p, and z be the impedance and equal to p/x (see Webster, loc. cit.). Let subscripts 1, 2, and 3 refer respectively to the values at the small end of the horn or in this case the vertex, the opening of the horn, and outside the opening at a point where the influence of the horn can be considered as vanishingly small. Assume the distance between p_2 and p_3 to be small compared with a wave-length and that Z_0 represents the impedance of the opening treated as a short tube or channel, the air escaping from a circular hole in an infinite plane. Then, by definition,

$$X_2 = \frac{p_2 - p_3}{Z_0} = \frac{Z_2 X_2 - p_3}{Z_0} = \frac{p_3}{Z_2 - Z_0}.$$

But according to Webster, equation (24)

$$X_2 = cp_1 + dX_1 = (cZ_1 + d)X_1,$$

where c and d are defined as follows:

$$c = \frac{-\sigma_2 x_1 \sin k(l + \epsilon_1 - \epsilon_2)}{\beta x_2 \sin k\epsilon_1 \sin k\epsilon_2}, \qquad d = \frac{\sigma_2 x_1 \sin k(l - \epsilon_2)}{\sigma_1 x_2 \sin k\epsilon_2}.$$

[1] Loc. cit.

σ_1 and σ_2 represent the cross sections at the small and large ends, respectively, $\beta = \rho a^2 k$, where ρ is density, a velocity of sound, and k the frequency divided by the velocity, x_1 and x_2 represent the distances from the vertex of the small and large ends respectively, and

$$l = x_2 - x_1,$$

$$\tan k\epsilon_1 = kx_1,$$

$$\tan k\epsilon_2 = kx_2;$$

$$\therefore X_1 = \frac{p_1}{Z_1} = \frac{X_2}{(cZ_1 + d)} = \frac{p_3}{(cZ_1 + d)(Z_2 - Z_0)}.$$

But since

$$Z_2 = \frac{aZ_1 + b}{cZ_1 + d} \quad \text{(Webster's equation (25))},$$

with

$$a = \frac{x_1}{x_2} \frac{\sin k(l + \epsilon_1)}{\sin k\epsilon_1}, \qquad b = \frac{\beta}{\sigma_1} \frac{x_1}{x_2} \sin kl,$$

then

$$p_1 = \frac{z_1 p_3}{(cz_1 + d)\left(\dfrac{az_1 + b}{cz_1 + d} - Z_0\right)},$$

and we have a relation between the exterior and vertex maximum excess pressures; and these pressures are proportional to the square roots of the respective intensities.

For the conical horn closed at the vertex,

$$z_1 = \infty,$$

and therefore

$$p_1 = \frac{p_3}{(a - cz_0)}.$$

Now Webster shows that, assuming the conductivity of the open end to be that of a short tube of conductivity c_0, ending in an infinite plane,

$$z_0 = \rho a^2 k^2 \left(\frac{k}{2\pi} i - \frac{1}{c_0}\right).$$

Hence the complete expression that will enable us to determine quantitative relationship between the incident sound intensity and that found at the vertex is

$$p_1 = \frac{p_3}{\dfrac{x_1}{x_2} \dfrac{\sin k(l + \epsilon_1)}{\sin k\epsilon_1} + \dfrac{\sigma_2 k x_1 \sin k(l + \epsilon_1 - \epsilon_2)}{x_2 \sin k\epsilon_1 \sin k\epsilon_2} \left(\dfrac{k}{2\pi} i - \dfrac{1}{c_0}\right)}$$

and if p is to be determined for the vertex, $x_1 = 0$,

$$p_1 = \frac{p_3}{\dfrac{\sin kx_2}{kx_2} + \dfrac{\sigma_2 \sin k(x_2 - \epsilon_2)}{x_2 \sin k\epsilon_2} \left(\dfrac{k}{2\pi} i - \dfrac{1}{c_0}\right)}. \qquad (1)$$

It can be shown (Professor Webster has shown this, as well as the derivation just given by the author, but has not published either) that Webster's theory may be modified by omitting the assumption of small cross section, and by assuming spherical waves, p being a function of r, the radial length, and the time, with the following equation:

$$p_1 = \frac{p_3}{\frac{\sin kr}{kr} + \frac{\sigma \sin (kr - \epsilon)}{r \sin k\epsilon}\left(\frac{k}{2\pi} i - \frac{1}{c_0}\right)}, \qquad (2)$$

where

$$\tan k\epsilon = kr.$$

For our case of conical horns (2) is the more accurate.

For the conductivity c_0, it is assumed that the open end is equivalent to a channel if length 0.6 of the radius, Thus

$$c_0 = \frac{\pi R^2}{0.6 \times R} = \frac{\pi R}{0.6}.$$

With this value of c_0 and using (2), computations were made and relative values of $(p_1/p_3)^2$ or relative values of the intensity, were secured. In computing, one must of course use the modulus of the fraction in the above equation. The results are plotted in the dotted curve in Fig. 2.

An additional point of the theory should be noted. Equation (2) indicates that the computed values depend upon the product kr which is proportional to the product of radial length and frequency. In Fig. 2, the experimental results were plotted with changing radial length, frequency remaining constant. From the foregoing statement it is obvious that precisely the same curve would have been obtained with varying frequency, radial length constant, the values of the ordinates depending only upon the product of radial length and frequency.

IV. Discussion of Results.

Extension of Results. Fig. 2.—Inasmuch as we are apparently justified, by the preceding theoretical considerations, in regarding Fig. 2 as showing the relationship between relative intensity and frequency, the following conclusions may be stated:

(*a*) The frequencies producing resonance are very closely integral multiples of the fundamental frequency.

(*b*) The resonance peaks broaden with increasing frequency.

(*c*) The intensity values for the minima, located between the resonance peaks, increase with increasing frequency.

(*d*) The conical horn gives a very considerable amplification for any frequency nearly the same as or greater than the fundamental frequency.

(The ordinate at the beginning of the curve is in fact no greater than the relative intensity without the horn at all.)

(*e*) The greater the frequency, the less the difference between maxima and minima relative intensities, the curve promising to approach one of smaller and smaller undulations, yet one representing a great amplification.

These conclusions indicate that if a horn, long compared to the wave-length of the least frequency involved, is used as a receiver, it promises amplification without highly marked resonance characteristics. Experimental tests of this conclusion have been made, and with satisfactory verification.[1]

Utilization of Results.—In the utilization of the results of the foregoing experimental and theoretical discussion it must be remembered that the existence of the optimum angle and its alteration with the order of the overtone, prevents the experimental curve in Fig. 2 from being regarded as quantitatively typical. In fact, if the horn ratio had been approximately 7 : 1 instead of 5 : 1, the second resonance peak would have been higher than the first. In general, since there exists an optimum angle, the relative values of the amplification at the various resonance frequencies must depend upon the angle of the horn. It would be advantageous to have information concerning the effect of various frequencies, with a variation of the ratio and length of the horn, for then one could ascertain just what effects any conical horn would have, acting as a receiver for any sound of known frequencies. It is believed that the present results furnish a partial guide in the design of a conical horn for any purpose, particularly by way of suggestion as to what trials should be made to secure the desired horn.

Physical Aspects.—An explanation in physical terms will assist in understanding the causes for the characteristics of the curve in Fig. 2. Assume for the moment that the length of the conical horn is short compared with the wave-length, and that the horn opening is a part of a plane wall. The horn would thus reflect the sound with approximately the same effect as any other equal area of wall surface, or surfaces supposed to be perfect reflectors. We would then expect to find the horn acting as a reflector and not as a collector. This reflecting function of the conical horn persists irrespective of its dimensions, though of course as the length of horn becomes comparable to the wave-length of the sound, the reflection cannot be considered as in the same phase from the entire inner surface, but each element of area is a source of sound in such phase and amplitude that combined with the oncoming sound will result in a

[1] Stewart, Phys. Rev., XIV., 2, 1919, p. 166.

zero value of the velocity potential over the wall surface. In the conical horn, then, reflection takes place throughout its entire length, this reflection being in all directions away from the inner surface just as if each element of that surface is a source of sound.

It is a consideration of this reflection which makes clear the reason for the lack of concentrating ability shown in that part of Fig. 2 to the left of the first resonance peak. Even when the radial length of the horn is approximately one-half of that for maximum resonance, the intensification produced by the horn is almost unnoticeable. In other words, the horn acts much like a plane reflector that subtends the same angle at the source and the energy is approximately the same at the vertex as if the opening were a hole in that plane.

Just as there is reflection from the walls of the incoming wave, so also there is reflection of the outgoing wave. In this latter reflection from the walls we find an explanation of the fact that the minima in Fig. 2 are not constant, showing zero amplification. Conditions for resonance demand reflection not only at a distant end but also at the opening where the incoming wave enters. In the conical tube, inasmuch as we have reflection of the outgoing wave throughout the entire length of the wall, there is always a certain amount of reflection at the point where the phase of the incoming wave and the just reflected portion of the outgoing wave are the same. There would thus be "resonance." According to this explanation there should be resonance for any frequency greater than the fundamental, for with every such frequency there is in at least one region of the inner surface a reflection of the outgoing wave with the above stated phase relations, thus furnishing the necessary conditions for resonance. Fig. 2 is in agreement with this conclusion in that the minima show a distinct amplification. As an illustration of the above explanation, suppose the horn has a length corresponding to the first minimum of Fig. 2. The reflection from the opening of the outgoing wave is opposite (approximately) in phase to the incoming wave. In a cylindrical tube this would be the cause of zero amplification. But in the conical horn there is reflection from the wall near the opening which, being reflection without change of phase, will be in phase (approximately) with the incoming wave. Resonance will therefore exist.

In this connection the writer wishes to emphasize that the usual view of the conical horn, *i.e.*, that it acts primarily as a concentrator, is misleading, and that the view of it as a resonator will, *in most applications be much more satisfactory and helpful.* Of course, both concentration and resonance are present and which predominates depends upon the dimensions of the horn and of the length of the wave.

In the usual theory, giving the resonating frequencies in conical tubes,[1] it is assumed that the incoming and outgoing waves are spherical within the tube and the results derived for the natural frequencies are approximately correct. But the assumption of sphericity is in effect an assumption of reflection from the walls throughout the length for otherwise there could not exist these imagined portions of spherical waves. In a portion of Webster's theory, to which reference has been made, plane waves are assumed in the case of a tube whose cross-section is always small, and spherical waves in the tubes not so limited, but there are applied hydrodynamic equations which permit reflections to exist. In the assumption of an approximately plane wave or a spherical wave reflections are virtually assumed, for these are necessary to maintain the stated shape of the wave. Thus it will be seen that existing theroies of the action of conical horns give approximate results for the natural frequencies because they distinctly assume reflection throughout the length of the walls, though without specific mention.

There are further points which should be considered from a phenomenon viewpoint, especially since our theory gives us no direct assistance toward an explanation. Two of these are the reasons for an optimum angle and the variation of the optimum with frequency and with the order of overtone. It is believed by the writer that the existence of the optimum angle is occasioned by the fact that the horn has a diameter not vanishingly small as compared with a wave-length and hence the vibrations are not merely radial. Obviously then, certain angles would be expected to give the optimum resonance. A theoretical investigation of these nonradial vibrations is now being made and a further discussion of the optimum angle will be postponed.

Errors.—The experimental results exhibited in this report should be interpreted in the light of the conditions under which they were undertaken. The results are and must be inaccurate because the conditions that would make possible accurate measurements cannot be had. In these experiments the horn is really a frustrum of a cone with a flux of energy through the "listening" tube attached at the several vertices. Inasmuch as the effect of the tube upon the intensity at the point of attachment in any case depends upon the area, length, damping, and resonance characteristics of the tube, it is not possible to establish experimentally a universally applicable statement of variation of relative intensification of the sound as dependent upon the length and angle of the horn and frequency of the sound used. It is to be observed, however, that, with the area and the length of the rubber tubing used in these

[1] See Rayleigh and also Barton, loc. cit.

experiments, the flux of energy would seem not to be great enough to destroy the value of the relative intensities obtained regarded as an approximation to the truth, that the damping in the tubing was sufficiently great to make the resonance in itself very small indeed, and that, in fact, inasmuch as the length of the tubing was kept constant and different frequencies are not compared, the small resonance effect which might exist did not seriously modify the variation of relative intensity with the angle and length of the horn. It is believed by the writer that, when all the physical conditions are given appropriate consideration, one is justified in accepting the results herein described as furnishing a fair idea of the relative intensities at the vertices of conical horns.

In conclusion, I wish to acknowledge the valuable assistance of Mr. Eugene M. Berry.

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THE ACTION OF ROENTGEN AND GAMMA RADIATIONS UPON THE ELECTRICAL CONDUCTIVITY OF SELENIUM CRYSTALS.

BY A. M. MCMAHON.

SYNOPSIS OF RESULTS.

Action of Gamma and Roentgen Radiations Demonstrated.—The increase in conductivity of selenium "cells" accompanying the absorption of X-rays has been found characteristic of individual crystals of metallic selenium. A similar effect upon the crystals has been observed to be produced by the absorption of the gamma rays of radium. The fact that a crystal fatigued to gamma rays is fatigued also to X-rays is evidence that the action in the two cases is similar. The indications are that the crystals are approximately twice as sensitive to gamma rays as to X-rays of the type used (L rays of tungsten).

New Electro-mechanical Properties.—Increase of mechanical pressure upon a crystal results in an increase of its sensibility both to X-rays and gamma rays. This is exactly the same result, qualitatively, as that secured by Brown in his study of the light effect (PHYS. REV., 4, p. 85).

Action of Separate X-ray Frequencies.—Unsatisfactory results obtained in the study of the strong lines of the L radiation of tungsten are thought to be due to the low intensities afforded by the X-ray spectrometer. Another method of attack is being planned.

Comparison of the Light, X-ray, and Gamma Ray Effects upon an Isolated Crystal.—Calculations made upon the basis of the energy absorbed indicate that a selenium crystal is most sensitive to gamma rays. The ratios of the change in conductivity to absorbed energy for exposures of one minute are 33.2, 58.5, and 108.9 $\left(\frac{\text{Mhos}}{\text{Joules}}\right)$, respectively.

I. INTRODUCTION

IN view of the established relationships between light, Roentgen and gamma rays, knowledge of their effects upon the electrical properties of crystals of metallic selenium assumes new interest. It has been shown[1] that the change in conductivity exhibited by these crystals on exposure to light depends upon the time, intensity, and character of the illumination. It is also known[2] that the sensibility (absolute ΔC) varies directly with the mechanical pressure to which the crystal is subjected. Other important facts concerning the action of visible radiations are available, but it is with those mentioned that this paper is mainly concerned. The effect of Roentgen rays upon films of metallic selenium has been investigated.[3] with the discovery that the form of the ΔC-time

[1] Brown, PHYS. REV., 33, p. 1.

[2] Brown, PHYS. REV., N.S., 4, p. 85.

[3] McDowell, PHYS. REV., 30, p. 474.

curves for this type of radiation is the same as that of those found for visible radiation, although the rate of recovery is much slower in the former case. The action of radioactive materials upon selenium "cells" has been known for some time,[1] although the separate action of the gamma rays had, within the knowledge of the writer, received no attention up to the time of the investigations reported upon in the present paper. It is thought that the continuation of this work should assist in relating properly the phenomena of conduction in solids with those of absorption, radiation, and atomic structure.

The experimental studies upon the action of X-rays, described in this paper, were initiated to secure information regarding the relation between the sensibility of these crystals and the frequencies of the strong lines in the characteristic L radiation of tungsten. Their change in conductivity on illumination has received a variety of theoretical treatments, among which that based upon a resonance hypothesis[2] offers considerable promise. According to this theory the electrons of the selenium atom whose radiation frequency corresponds, or nearly corresponds, to the frequency of the exciting agent, are loosened from their atomic bonds temporarily, and, in addition to the normal number free at any instant within the crystal, as indicated by the conductivity in the dark, contribute to the total current resulting from the application of a potential. A region of maximum sensibility has been found[3] in the red, and indications of a maximum in the ultra-violet. It was thought that if a region of maximum sensibility could be found within the range of X-ray frequencies the resonance theory would be much strengthened, and important evidence concerning the structure of the selenium atom obtained. Other experiments, incidental to this purpose, which have brought out a significant relation between the X-ray, gamma ray, and light effects, as regards the dependence of sensibility (absolute ΔC), upon mechanical pressure, constitute an important part of this report.

The studies herein described were made upon a single crystal selected from a lot prepared by Dr. W. E. Tisdale in the spring of 1916. They were of the hexagonal[2] variety secured by distillation in vacuo at 185° C. Investigations were in air at barometric pressure. The dimensions of the crystal chosen were roughly $3.2 \times .2 \times .1$ mm. It was mounted between carefully polished platinum electrodes, the mechanical pressure necessary being supplied by weights. In every case the direction of the rays was parallel to the long axis of the crystal.

[1] Brown PHYS. REV., 26, p. 273.

[2] Pfund, PHYS. REV., 28, p. 324.

[3] Sieg and Brown, PHYS. REV., N.S., 4, p. 507.

II. The Action of X-rays.

Preliminary measurements upon the change of conductivity produced by single lines in the L radiation of tungsten indicated that the effect, if discernible with the apparatus used, is very small. A simple experiment was then planned and carried out to ascertain whether the addition of mechanical pressure would, as is the case when the exciting agent is visible light, produce an increase of sensibility.

The crystal was mounted in a metal box which was grounded. The leads were carried through lead tubing to a Wheatstone bridge of high resistance arms, situated in an adjoining room. All of this was found necessary to eliminate the electrostatic effects which otherwise resulted from the presence of the high potential transformer actuating the Coolidge tube furnishing the X-rays. In this experiment the total radiation from the tube, operating at 48.0 kv. and 3.5 amps. heating current, was used. It was found difficult to maintain the constancy of the input with better than 10–15 per cent. accuracy. Hence the distribution of points in Fig. 1. A heavy pendulum operating contact keys controlling

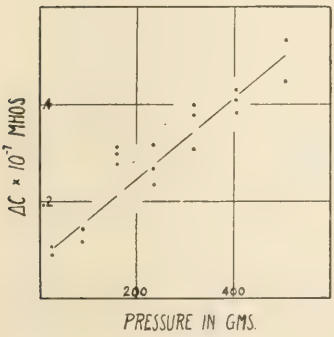

Fig. 1.

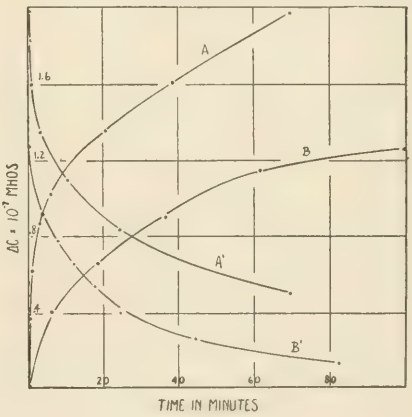

Fig. 2.

Fig. 1. The effect of change in mechanical pressure upon sensibility to X-rays.

Fig. 2. Excitation and recovery curves—X-rays. *A*, Excitation. Pressure 320 gms. Initial Conductivity 4.8×10^{-7} Mhos. *A'*, Recovery. *B*, Excitation. Pressure 320 gms. Initial Conductivity 4.6×10^{-7} Mhos. *B'*, Recovery.

the primary circuit of the transformer provided the means of keeping the time of exposure constant (one second), for different pressures. Time for recovery was allowed between observations. Brown and Clarke's method[1] was used in observing the change in conductivity. The results, plotted in Fig. 1, indicate an unmistakable increase of sensibility with pressure, similar to that found by Brown to exist when the exciting

[1] Brown and Clarke, Phys. Rev., 33, p. 53.

agent is light. Two other crystals studied in the same manner gave data which bear out this conclusion.

Following a procedure made obvious by these results, measurements were made anew upon five of the principal lines in the L radiation of tungsten (λ's-1.47, 1.27, 1.24, 1.09, 1.06Å). Five hundred grams pressure was applied to the crystal—as much as was deemed advisable in view of the danger of fracturing. I am indebted to Dr. Elmer Dershem for the use of one of his X-ray spectrometers in these experiments as well as for initiation into the technique of its manipulation. Up to the present time, however, attempts to secure the effects of separate lines in this manner have been unsuccessful. Experiments are being considered which may solve the intensity difficulty, for this is thought to be the main reason for the failure.

It is very readily demonstrated that the total radiation from a Coolidge tube, operating upon a moderate energy input, produces a change in conductivity. Fig. 2 gives the results of two series of observations made upon the excitation and recovery characteristics of the crystal. Two additional experiments prove that these data come from changes in the conductivity of the crystal due to the absorption of X-rays. The apparatus was the same as described for the pressure-ΔC experiment. First, with no heating current flowing in the Coolidge tube filament, but with the customary 48.0 kv. potential applied, no galvanometer deflection was observed—*i.e.*, the balance of the bridge was maintained. Second, the introduction of a sheet of lead into the path of the rays from the target to the crystal resulted in a smaller total change than that produced in the absence of the lead. In making the measurements from which the data represented in Fig. 2 were computed a null method was used. The excitation and recovery of the crystal are so very much slower than the action of the galvanometer used that it was practical to change the balancing arm of the bridge by a convenient amount, usually 100 ohms, and observe with a watch the time required for the balance to be reattained.

III. The Action of Gamma Rays.

It was found time conserving to carry on experiments with gamma rays along with the X-ray studies, and in consequence of this practice a very interesting relation was discovered to exist between the gamma ray and X-ray effects, which, however, might have been predicted. In the first experiments with the crystal whose properties are here discussed gamma rays were used. When these were finished X-ray excitation and recovery curves were secured, six hours having been allowed for recovery. The results of the latter are given by curves B and B' of Fig. 2. By

comparison with the characteristics of other crystals from the same lot it was thought that the equilibrium value of the conductivity change, indicated by B, was too low. More time (48 hrs.) was allowed for recovery and curves A and A' secured, which show an equilibrium value comparing favorably with those secured for other crystals. Unquestionably, therefore, a crystal fatigued to gamma rays is also fatigued to X-rays, a result which is in accord with the view that these radiations are of a similar nature.

The effect of the gamma rays was obtained by lowering a thin glass vial containing a milligram of radium into proximity with the crystal through a hollow lead cylinder with walls a millimeter thick to absorb the alpha and beta rays. In these experiments the drift of the galvanometer spot was noted at ten second intervals and later translated into change in conductivity by proper calibration of the bridge. This alteration in method was found necessary in order to differentiate clearly between the effect of the gamma rays and the initial variation common in the balance of the bridge. Much more data than can be given here were taken to make sure of this point. It is thought that the action of

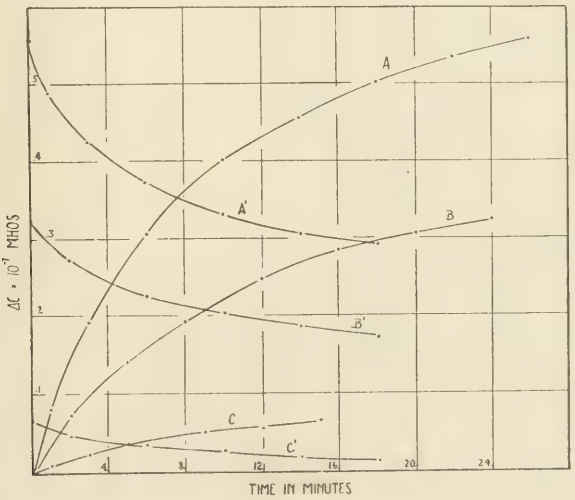

FIG. 3.

Excitation and recovery curves—gamma rays. A, Excitation. Pressure 320 gms. Initial Conductivity 5.53×10^{-7} Mhos. A', Recovery. B, Excitation. Pressure 216 gms. Initial Conductivity 5.08×10^{-7} Mhos. B', Recovery. C, Excitation. Pressure 24 gms. Initial Conductivity 1.25×10^{-7} Mhos. C', Recovery.

the gamma rays is established beyond question. Fig. 3 gives curves secured with the same crystal as before for three different pressures. The error due to variations in the balance of the bridge is probably less

than 1 per cent. for the maximum values of ΔC in curves A and B. The increase of sensibility with increase of mechanical pressure is unmistakable.

IV. A Comparison of the Light, X-ray, and Gamma Ray Effects.

It is of great importance to compare the X-ray, gamma ray, and light effects from the standpoint of absorbed energy. Although the data thus far available do not permit of direct comparison, it is possible by making certain assumptions to base a calculation upon them which leads to an interesting result. It is hoped that the additional measurements necessary for a more satisfactory comparison may be accomplished in the near future. The present calculations were made as follows.

The energy of the Roentgen rays was secured from the input to the Coolidge tube and the value for the efficiency of production at 48.0 kv. found by Rutherford.[1] The energy of the gamma rays was found from their known heating effects. An observation was made upon the change in conductivity undergone when light of .7 μ and known intensity was incident upon the crystal. Assuming the absorption law $I = I.e^{-\mu x}$ for Roentgen and gamma rays, and considering that all of the light entering the crystal must have been absorbed (the crystals are not transparent), we get the following results:

Radiation.	E_i = Incident Energy $\frac{\text{Ergs}}{\text{Sec.}}$.	E_a = Absorbed Energy $\frac{\text{Ergs}}{\text{Sec.}}$.	ΔC in 1 Min.
Gamma rays (Ra) . .	6.57×10^{-3}	3.74×10^{-4}	$.08 \times 10^{-7}$
X-rays (L of W)	2.60×10^{-2}	1.06×10^{-2}	$.62 \times 10^{-7}$
Light (.7 μ)	1.84×10^{-3}	1.19×10^{-3}	$.15 \times 10^{-7}$

Benoist's relation between absorption and atomic weight was used in obtaining the X-ray absorption constant, $\mu = 1.64$. The assumption of Rutherford's relation between density and absorption was necessary to secure the gamma ray absorption constant, $\mu = .184$. The distribution of X-ray, gamma ray, and light energy was considered uniform.

From these results and the intensity law,[2] $C = k\ I^{1/2}$, the change in conductivity when $E = 2.60 \times 10^{-2}$ was calculated for each type of radiation. The absorbed energy was found, also, and the ratio, $\Delta C/Ea$, formed, viz.:

Radiation.	ΔC in 1 Min.	E_a in $\frac{\text{Ergs}}{\text{Sec.}}$.	$\frac{\Delta C}{E_a}$.
Gamma rays	$.16 \times 10^{-7}$	1.47×10^{-3}	108.9×10^{-7}
X-rays	$.62 \times 10^{-7}$	1.06×10^{-2}	58.5×10^{-7}
Light	$.56 \times 10^{-7}$	1.69×10^{-2}	33.2×10^{-7}

[1] Rutherford, Phil. Mag., 30, p. 361.

[2] Nicholson, PHYS. REV., N.S., 3, p. 1.

Although the figures resulting from these computations are in the nature of approximations it is hoped that their order of magnitude may prove suggestive of the possibilities in this line of work.

The conclusion of the greater sensibility to gamma rays seems quite certain, however. For if instead of the above square-root intensity law we assume the linear law[1] for faint illumination, the comparison is even more favorable to the gamma rays. Discussion of just what this apparent difference may mean is postponed until a more complete experimental study can be made.

In conclusion I wish to express my appreciation of the interest the staff of the Department of Physics of the University of Iowa has taken in these problems. Especially do I wish to thank Dr. F. C. Brown for his suggestions and encouragement throughout the progress of the work.

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[1] Brown and Sieg, Phys. Rev., N.S., 2, p. 487.

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THE BLACKENING OF A PHOTOGRAPHIC PLATE AS A FUNCTION OF INTENSITY OF LIGHT AND TIME OF EXPOSURE.

BY P. S. HELMICK.

SYNOPSIS.

Blackening of Photographic Plates as a Function of Light Intensity, Wave-length and Exposure.—Three emulsions, a slow one (Seed 23), a rapid one (Seed 27*X*) and a very rapid one (Seed Graflex) were coated on plate glass and illuminated for various lengths of time with various intensities of monochromatic light of wave-length 450, 550 and 650 $\mu\mu$. The results are given in curves and are found to be well represented, for a given emulsion and wave-length, by the *empirical equation for blackening*, $D = (1/a) \log [b - (b - 1)e^{-ct}]$, where $\log c = A + B \log I + c \log^2 I$, and a, b, A, B and C are constants. A graphical method of determining these constants employing a monographic chart is described. Schwarzschild's exponent p is shown to be equal to $(B + C \log I)^{-1}$, and is found to vary from 0.7 to 1.95 according to the emulsion, wave-length and intensity. In using the *photographic method of measuring the intensity of a source of light*, it is obviously important to determine the constants of the equation for the blackening of the emulsion for the proper wave-length. The method of doing this is explained in detail.

THE object of this research has been the expression of the blackening of a photographic plate as a function of the intensity of monochromatic light and time of exposure to light, other factors being constant.

Such an expression finds practical application in photometry of monochromatic sources. By exposing a plate to a light source of unknown intensity for any known length of time, measurement of the resulting blackening will enable the unknown intensity of light to be calculated.

APPARATUS AND MATERIALS.

The plates used in the work were coated on special plate glass, and the variations of density due to unevenness of coating were of the order of 1 per cent. Three emulsions were used: "*A*" a slow emulsion (Seed 23), "*B*" a rapid emulsion (Seed 27*X*), and "*C*" a very rapid emulsion (Seed Graflex).

The source of light was a Hilger Monochromatic Illuminator. Light from a constant-voltage tungsten lamp was focussed upon the collimating slit of the illuminator, and was again focused upon the second slit of the instrument, before emerging into the exposing box.

Variations in intensity of light were obtained by varying the distance

between the plate and the second slit of the illuminator, assuming the inverse square law, and applying the necessary correction for finite width of source.[1]

Intensities were measured in watts/sq. meter by a thermopile calibrated against a standard lamp whose radiation was certified to by the Bureau of Standards.

Exposures were made by cutting one of the 5 x 7 plates into 30 strips, and exposing 12 of these, the remaining strips being used to obtain the amount of fog, as explained later. Three wave-lengths of light were used: 450 $\mu\mu$, 550 $\mu\mu$, and 650 $\mu\mu$. A hand-operated shutter was mounted in front of the slit of the illuminator, and by listening to a telephone receiver clicking seconds, exposures consisting of any integral number of seconds could easily be made.

Plates were developed at a practically constant temperature in a developer compounded after Brush's formula.[2] The maximum variation in temperature during development of the plates which make up one of the sets of curves shown later on was never more than 1°.8 C., generally it was only a few tenths of a degree. A time of development of 6 minutes at 23° C. was selected as standard. This variation in temperature was corrected for by the use of Watkins' formula[3]

$$r = (s_1/s_2)^{\frac{10}{t_2-t_1}},$$

where r = temperature coefficient, t_1 = low temperature, t_2 = high temperature, s_1 = time of development at t_1, and s_2 = time of development at t_2.[4]

The density of a photographic plate is defined as Log_{10} o, where o, the opacity, is the ratio of the incident to the transmitted light. Densities were measured in a modification of Lemon's spectro-photometer,[5] in which the prism is replaced by two mirrors inclined to one another so as to reflect two beams of light into the observing telescope. The density corresponding to an angle θ between transmitting planes of the nicols when rotated so as to cut down one light beam to the same intensity which the interposed plate of unknown density cuts down the other light beam is given by the expression $\mathrm{Log}_{10}\,\mathrm{Sec}^2\,\theta$.

[1] See Bull. Bur. Standards, 3, 81; 1907.

[2] PHYS. REV., 31, 243; 1910.

[3] The A. Photographer and Photo. News, London, May 17, 1910.

[4] According to Watkins, the value of "r" for any hydroquinone developer is 2.25, so this value was accordingly taken for the "r" of the hydroquinone developer used in this work. For example, if the temperature were 22° C., the time of development allowed was given by the solution of the equation

$$2.25 = \left[\frac{S_1}{6}\right]^{\frac{10}{33-22}}.$$

[5] Astrophys. Journ., 39, 204; 1914.

Densities were measured with light of 535 $\mu\mu$, obtained by placing two "monochromatic" color filters in front of the observing telescope. Density readings on a single plate were repeated until the probable error of the mean was reduced to a very small value.

Brand A.

Wave-length 550 $\mu\mu$. Distance 226.2 cm. 1.53×10^{-4} watts/sq. m. Temperature 22°.0. Time Devel. $6^m\ 30^s$.

Plate.	Time.	D	Plate.	Time.	D
1500		0.35	1514		0.29
1501	15^m	6.31	1515	15^s	0.32
1502		0.39	1516		0.26
1503	8^m	4.03	1517	8^s	0.25
1504		0.37	1520		0.25
1505	4^m	2.88	1521	4^s	0.27
1506		0.42	1522		0.27
1507	2^m	1.91	1523	2^s	0.28
1510		0.34	1524		0.27
1511	1^m	1.35	1525	1^s	0.26
1512		0.31	1526		0.24
1513	30^s	0.58	1527		0.28

The preceding table is a sample of one set of exposures. All of the separate small plates were cut from the same 5 x 7 plate. The small plates were numbered to show how they were cut with respect to the original plate. For example 1513 lay adjacent to 1512, 1514, 1503 and 1523 in the original plate. The even numbers were not exposed, and were used to obtain the fog. These unexposed fog strips were developed along with the exposed plates in order to find the blackening of the exposed plates which was not due to exposure to light. This fog density was then subtracted from the densities of the exposed plates in the calculations and curves.

The plates were of brand "*A*," exposed to light of wave-length 550 $\mu\mu$ at a distance of 226.2 cm. from the slit of the monochromatic illuminator, the flow of energy on the plate being 1.53×10^{-4} watts/sq. m. The plates were developed at a temperature of 22°.0 for a time of $6^m\ 30^s$.

The table gives the plate number, the time of exposure, and the mean of the density determinations of the plate.

Theoretical.

In 1890, Hurter and Driffield[1] developed an equation of the following form for the density D of a photographic plate exposed for a time t to light of constant intensity I:

$$D = \alpha \operatorname{Log}_e [\beta - (\beta - 1)e^{-\gamma It}],$$

[1] Journ. Soc. Chem. Ind., 9, 455; 1890.

where α, β, and γ are parameters dependent upon the character of the plate and the developer, $-\beta$ is equal to the opacity of the unexposed plate. They assumed that the amount of silver made developable by the light was proportional to $I \cdot t$, and that only the light absorbed by the unaltered silver particles contributed to the building up of the latent image.

This expression would lead to the so-called "Reciprocity Law" of Bunsen and Roscoe[1] ($I \cdot t$ equals constant for equal blackening) which Abney,[2] Miethe,[3] Eder,[4] Michalke and Schiener, Schwarzschild,[5] Kron,[6] Lemon,[7] and others have shown to be only a very rough approximation to the facts.

However, for a constant intensity of light, the expression gives a very good relationship between the density of the plate and the time of exposure, as both Hurter and Driffield, and Sheppard and Mees,[8] have shown.

In this work the equation of Hurter and Driffield is made use of in the following way:

Given a set of curves showing the relationship between density and time of exposure for intensities of light $I_1 = k_1$, $I_2 = k_2$, $\cdots$, $I_n = k_n$.

We will rewrite the equation of Hurter and Driffield in the form:

$$D = 1/a \operatorname{Log}_c [b - (b - 1)e^{-ct}].$$

Then for the curves $I_1, I_2, \cdots, I_n$, we can find values of the parameters a, b, c_1, c_2, $\cdots$, c_n, respectively which will give us this set of n curves. But c is a function of I, and if we can find this function, we can write our equation in the form:

$$D = 1/a \operatorname{Log}_c [b - (b - 1)e^{-f(I)\times t}]$$

and thus the density is completely determined in terms of the intensity of light and time of exposure.

Hurter and Driffield gave no hint as to the method of determination of the parameters a, b, and c, and Sheppard and Mees give a method applicable only when $a = 1$. Two methods are here developed which may be used to find the values of these parameters from a given experimental curve.

[1] Ann. der Phys., 117, 538; 1862.

[2] Phot. Journ., Oct., 1893.
Journ. Camera Club, 8, 46.

[3] Inaug. Diss. Gottingen, 1899.

[4] Handbuch, Bd. 2.
Jahrbuch, 1899, 457.

[5] Phot. Corr., 1899. Beitrage zur Phot. Photem. d. Gestirne. Astrophys. Journ., 11, 89; 1900.

[6] Ann. der. Phys., 41, 755; 1913.

[7] Loc. cit.

[8] Investigations on the Theory of the Photographic Process. Longmans, p. 210.

Assuming that the experimental curve is of the form given above, select four points, (t_1, D_1), (t_2, D_2), (t_3, D_3), and (t_4, D_4), so that $t_2 = t_1 + \Delta t$, $t_3 = t_1 + 2\Delta t$, $t_4 = t_1 + 3\Delta t$.

Then

$$e^{a(D_1 - D_2)} = \frac{b - (b-1)e^{-ct_1}}{b - (b-1)e^{-ct_1}e^{-c\Delta t}},$$

$$e^{a(D_1 - D_3)} = \frac{b - (b-1)e^{-ct_1}}{b - (b-1)e^{-ct_1}e^{-2c\Delta t}},$$

$$e^{(D_1 - D_4)} = \frac{b - (b-1)e^{-ct_1}}{b - (b-1)e^{-ct_1}e^{-3c\Delta t}},$$

$$\frac{b - (b-1)e^{-ct_1}e^{-c\Delta t}}{e^{a(D_2 - D_1)}} = \frac{b - (b-1)e^{-ct_1}e^{-2c\Delta t}}{e^{a(D_3 - D_1)}} = \frac{b - (b-1)e^{-ct_1}e^{-3c\Delta t}}{e^{a(D_4 - D_1)}},$$

$$\frac{e^{a(D_3 - D_1)} - e^{a(D_4 - D_1)}}{e^{a(D_2 - D_1)} - e^{a(D_3 - D_1)}} = e^{-c\Delta t},$$

(1) $$c = (1/\Delta t)\ \mathrm{Log}_e \frac{e^{aD_2} - e^{aD_3}}{e^{aD_3} - e^{aD_4}},$$

(2) $$(e^{aD_1} - e^{aD_2})(e^{aD_3} - e^{aD_4}) = (e^{aD_2} - e^{aD_3})^2,$$

(3) $$b = \frac{e^{aD} - e^{-ct}}{1 - e^{-ct}}.$$

These last three equations are sufficient to determine a, b, and c.

The practical difficulties connected with this method are quite material,

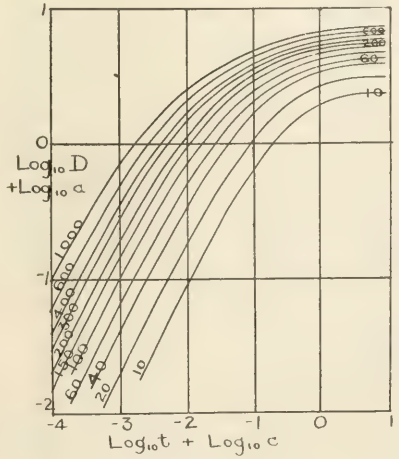

Fig. 1.

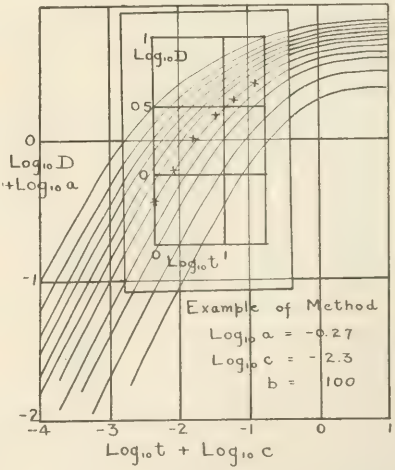

Fig. 2.

so a simple graphical method was accordingly developed.

A family of 11 curves

$$D = 1/a\ \mathrm{Log}_e\,[b - (b-1)e^{-ct}]$$

was plotted as shown in the figure, with b a parameter ranging in value from 10 to 1,000, as in Fig. 1.

Log aD was plotted along the y-axis, and Log ct along the x-axis. The experimental curve was plotted upon tracing paper with Log D and Log t as ordinate and abscissa, respectively. The tracing paper was shifted over the family of curves preserving parallelism of axes, until it coincided with one of the family. b was determined by the particular curve of the family which gave coincidence, and the a and the c were given by the intercepts of the axes of the experimental curve upon the axes of the family of curves. (Fig. 2.)

In actual practice, especially for large ranges of times of exposures, it was not difficult to find the position of the tracing paper which gave the optimum fit of the experimental curve to one of the theoretical curves,—*i.e.*, that the values of a, b, and c are unique.

Log c was next expressed as a function of Log I by Campbell's "Method of the Zero Sum,"[1] assuming that

$$\text{Log}_{10} c = A + B \text{ Log}_{10} I + C \text{ Log}_{10}^2 I.$$

This relationship was selected because the curvature of the Log c − Log I curve was very small, and because the equation readily lends itself to methods of adjustment of constants, and to the determination of the exponent "p" in Schwarzschild's "Law."[1] ($I \cdot t^p$ equals constant for equal blackening.)

In one case, namely the set of plates A–650, the curvature was more marked than in the other cases, and a term involving $\text{Log}_{10}^3 I$ was added to the expression.

The following values of the parameters were obtained by this method:

Plate.	a	b	A	B	C	D
A–450	0.63	200	3.99	1.95	0.10	
A–550	0.63	200	0.10	1.34	0.06	
A–650	0.40	200	−1.37	2.56	0.91	0.16
B–450	1.41	200	5.92	2.57	0.16	
B–550	1.12	200	−0.94	0.37	−0.12	
B–650	1.26	200	−1.00	1.38	0.11	
C–450	1.58	400	4.46	1.94	0.10	
C–550	1.12	100	0.51	1.00	−0.03	
C–650	1.58	200	−0.26	1.87	0.15	

$$\text{Log}_{10} c = A + B \text{ Log}_{10} I + C \text{ Log}_{10}^2 I + D \text{ Log}_{10}^3 I.$$

[1] Phil. Mag., 39, 177; 1920.

[2] Loc. cit.

In Fig. 3 the $\text{Log}_{10} c - \text{Log}_{10} I$ curves are plotted for the 9 different sets of plates. Points show values of "c" determined graphically.

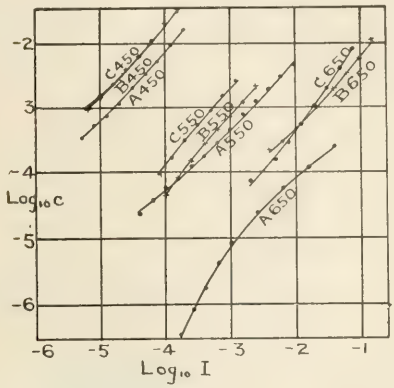

Fig. 3.

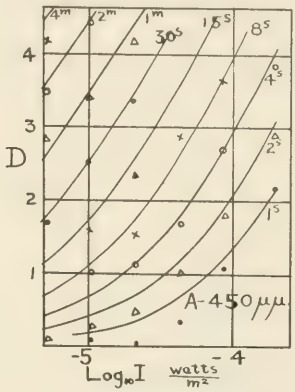

Fig. 4.

Curves show values of "c" computed from

$$\text{Log}_{10} C = A + B \text{ Log}_{10} I + C \text{ Log}_{10}^2 I.$$

Figs. 4 to 12 show values of blackenings computed from the above

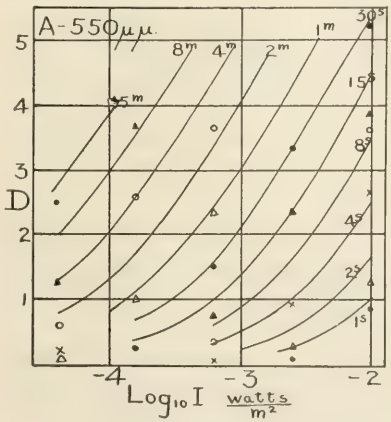

Fig. 5.

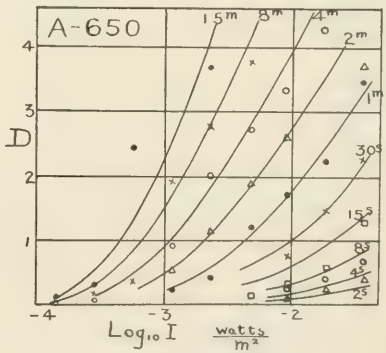

Fig. 6.

parameters, and the points represent the experimentally determined values.

The "p" of Schwarzschild's so-called "Law" can be very easily obtained from the equation which the writer has used to express the relation between the density of a plate and the intensity of light and time of exposure.

Schwarzschild states that the product $I \cdot t^p$ must be constant for equal

blackening, but Kron[1] and Lemon[2] have shown that p is a function of the intensity of light.

Expressing the blackening of a plate in the form

$$D = 1/a[\text{Log}_e b - (b - 1)e^{-ct}],$$

where

$$\text{Log}_{10} c = A + B \text{Log}_{10} I + C \text{Log}_{10}^2 I,$$

then $c \cdot t$ must be constant for equal blackening, or

$$t \cdot 10^{A+B \text{Log}_{10} I + C \text{Log}_{10}^2 I} = \text{constant.}$$

or

$$t \cdot I^{B+C \text{Log}_{10} I} = \text{constant.}$$

So $(B + C \text{Log}_{10} I)^{-1}$ is equivalent to the "p" of Schwarzschild's expression.

The following extreme values of p were computed in this manner, and show the variation in the value of p with the intensity of light.

Plate.	Range of Log_{10} Intensity.			Range of p.		
A–450.....	−3.7	−5.3		1.09	1.01	
A–550.....	−2.0	−4.4		1.22	1.06	
A–650.....	−1.4	−3.8	(−2.92 Max.)	1.59	1.36	(1.23 Max.)
B–450.....	−3.8	−5.2		1.95	1.72	
B–550.....	−2.6	−4.0		0.68	0.85	
B–650.....	−0.8	−2.4		1.30	1.13	
C–450.....	−3.8	−5.2		1.58	1.44	
C–550.....	−2.9	−4.1		1.05	1.08	
C–650.....	−1.1	−2.7		1.71	1.48	

Procedure in Using Method for Photometric Work.

"Calibration" of Plates.

1. Plates are exposed to a known intensity of monochromatic light for various times of exposure and a Log D − Log t curve plotted. Values of the constants a and b are determined from this curve with the aid of the monographic chart.

2. Plates are exposed for constant times of exposure to various known intensities of the monochromatic light and a Log D − Log t curve plotted as in 1. Values of c as a function of I are now determined with the monographic chart.

3. Log c is plotted against Log I.

[1] Loc. cit.

[2] Loc. cit.

Determination of Intensity.

1. A plate is exposed for any length of time to the monochromatic light of unknown intensity.

2. Knowing the density of the plate, the time of exposure, and the

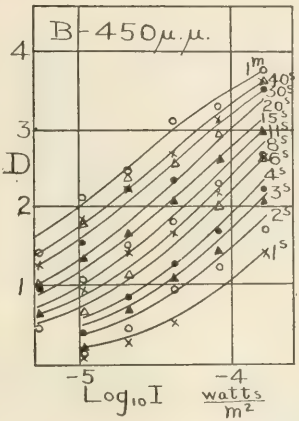

Fig. 7.

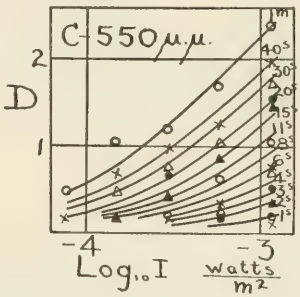

Fig. 8.

values of the constants a and b, the value of c for the particular intensity of light is determined from the monographic chart.

3. From the determined value of c, the value of I is found from the Log c − Log I curve.

Some General Considerations.

In plotting photographic blackening curves, the standard practice has always been to plot density of plate against logarithm of time of exposure, intensity being a parameter. In the light of the foregoing developments,

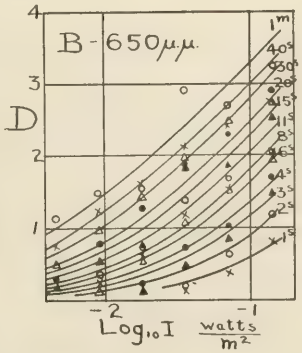

Fig. 9.

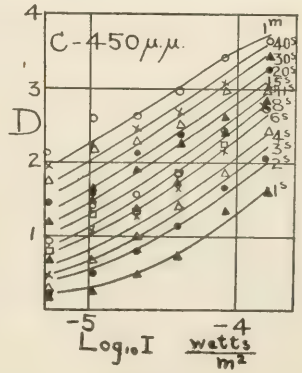

Fig. 10.

as a and b are constant for a given set of curves, it would be more logical to plot logarithm of density against logarithm of time,—for in that case the result would consist of a family of parallel curves. The actual drawing of such a family would be greatly facilitated, because the curves would be identical in shape. In this work however, the curves have

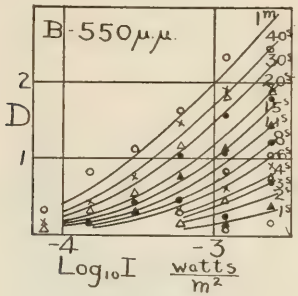

Fig. 11.

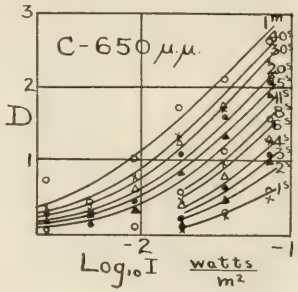

Fig. 12.

been plotted according to the standard practice in order that they may be comparable to previously published curves.

A set of exposures such as has been made throws much light upon the characteristics of fast and slow plates.

Consider first the relationship between brand of plate and blackening, with equal times of exposures and intensities of light, other factors being constant. In general, the curves of the slower plates are steeper than those of the faster ones.

The result is, that for a small intensity of light, the fast plate attains a greater density for a given time of exposure than does a slower plate. But for a longer time of exposure, the slower plate may attain a greater density than does the fast one. It is common knowledge among photographers that the longer the exposures, the less the difference that can be noticed between fast and slow plates.

The fact that the curves of the slow plates are steeper than those of the fast ones accounts for the greater contrast of a slow plate as compared to a fast one. With time of exposure constant, a small difference of intensity of light on the slow plate means a much greater difference in density in the slow plate than in the fast one.

The effect of wave-length of light upon the blackening, with intensity and time constant, is of course, very apparent, but the relation between slope of curves with wave-length of light is not marked. In fact, Leimbach[1] claims, though others have questioned his conclusions, that the slope, or "gradation" is independent of the wave-length. Upon closer

[1] Zeit. wiss. Phot., 7, 157 u. 181; 1909.

inspection, the writer's curves show a tendency to have the least gradation for that wave-length where the plate has its greatest sensibility, as Abney,[1] Stark[2] and others have found.

Certain generalizations can also be made with regard to the Log c − Log I curves. First, the curves depart more or less from a straight line. If Schwarzschild's expression were valid,—*i.e.*, $I \cdot t^p$ = constant for equal blackening, the Log c − Log I curves would be straight lines, as may be proved from the expression $p = (B - C \operatorname{Log}_{10} I)^{-1}$. The curves may therefore be regarded as giving some indication of the amount of error which is involved in assuming Schwarzschild's expression.

If Bunsen and Roscoe's "Law" held,—*i.e.*, $I \cdot t$ = constant for equal blackening, the curves would be straight lines with slope equal to unity, or c would equal I (Hurter and Driffield's assumption). The curves show that this is only a very rough approximation.

Conclusions.

Two methods of obtaining the mathematical equation of the blackening of a photographic plate as a function of intensity of monochromatic light and time of exposure have been pointed out.

Using the method of the monographic chart, the blackening equations of three different brands of plates have been calculated for three different wave-lengths of light. In these nine different cases, there is fairly good agreement between the experimental and the calculated values.

It is a pleasure to acknowledge the encouragement received from the department of physics of the State University of Iowa, and particularly from Professors L. P. Sieg, and H. L. Dodge, the latter now of the University of Oklahoma.

Ross[3] has pointed out an apparent inconsistency in the assumptions underlying the Hurter and Driffield equation made use of in the formulae above, and as a substitute develops the equation

$$D = d_m \left[1 = \frac{1}{n} \sum_{s=0}^{s=n-1} e^{-r^s k I t} \right].$$

D is the density of the plate, d_m the density for infinite exposure, I the intensity of light (assumed monochromatic, t the time of exposure, and k, r, and n are certain parameters. A method is given of determining n if d_m can be found by experiment, if arbitrary values assigned to r and k. The expression will hold only for one intensity of light unless the Reciprocity "Law" ($I \cdot t$ = constant for equal blackening) be assumed.

[1] Treatise on Photography, 10th Ed. Longmans, p. 413.

[2] Ann. der Phys., 35, 474; 1911.

[3] Journ. Optical Soc. of Am., 4, 261; September, 1920.

Ross' formula can be made the basis of expressing the blackening of a plate exposed to monochromatic light as a function of intensity of light and time of exposure according to the graphical method already outlined above if the formula is written in the form

$$D = \frac{1}{a}\left[1 - \frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r*ct}\right],$$

and families of Log aD − Log ct curves are plotted with n and r as parameters.

A set of experimental Log D − Log t curves is obtained (I having a different value for each curve), and a particular Log aD − Log ct curve is chosen so that the n, r, and a will give a good fit to each of the Log D − Log t curves by allowing c to vary. The c and the a are found from the x and the y-intercepts of the axes of the Log D − Log t curves on the axes of the Log aD − Log ct axes, and the n and the r are determined by the particular Log aD − Log ct curve chosed to give the best fit. If c be next expressed as a function of I, we have a mathematical expression for the blackening of a photographic plate as a function of the intensity of monochromatic light and time of exposure.

The possible advantages of a graphical means of determining the parameters of Ross' equation seems to lie in the fact that by a graphical method the reader can easily judge the accuracy of fit of the experimental to the theoretical curve, and can make the curves fit particularly well at any desired point, that he can find at once the values of the parameters r, n, and k, and that he is not limited to using one particular value of r or k.

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THE INTRINSIC INTENSITY OF LIGHT TRANSMITTED THROUGH A SLIT AS A FUNCTION OF THE WIDTH AND DEPTH OF THE SLIT AND OF THE WAVE LENGTH OF THE LIGHT

BY

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Introduction

It appears that very little work has been done on the absorption of light by a narrow, deep slit. This offers a very interesting problem, for even a simple observation indicates that there is apparently a much stronger absorption than one might anticipate. In addition there is a strong polarization of the light. This latter phenomenon is very striking, but it has not been studied quantitatively in this first work. Rayleigh[1] has treated theoretically the polarization of light transmitted by a narrow slit in an infinitely thin, opaque screen. That work serves in this present instance principally to enable us to define a "deep" slit. A "deep" slit shall here be considered as one which causes polarization of incident natural light such that the electric vector parallel to the length of the slit is greater than that perpendicular to the slit. In this sense it is well nigh impossible to attain to a thin slit. Even a thin scratch in a very thin silver film on glass constitutes a "deep" slit. Stewart[2] noticed strong polarization and absorption by a narrow slit in a thick screen in some experiments performed in another connection. It was thought a matter of interest, and of some importance to investigate the intensity of light transmitted by a deep slit as a function of the width and depth of the slit, and of the wave length of the light. Further work should be done on the variation of the material constituting the jaws of the slit.

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[1] Rayleigh. Roy. Soc. Lond. Proc. A, 89, p. 194, 1913–14.

[2] G. W. Stewart. Abstract. Phys. Rev. 5, p. 73, 1915.

APPARATUS AND METHOD OF PROCEDURE

A sketch of the arrangement of the apparatus used is shown in Fig. 1. Light from a tungsten strip lamp was focused on S_1 of

FIG. 1

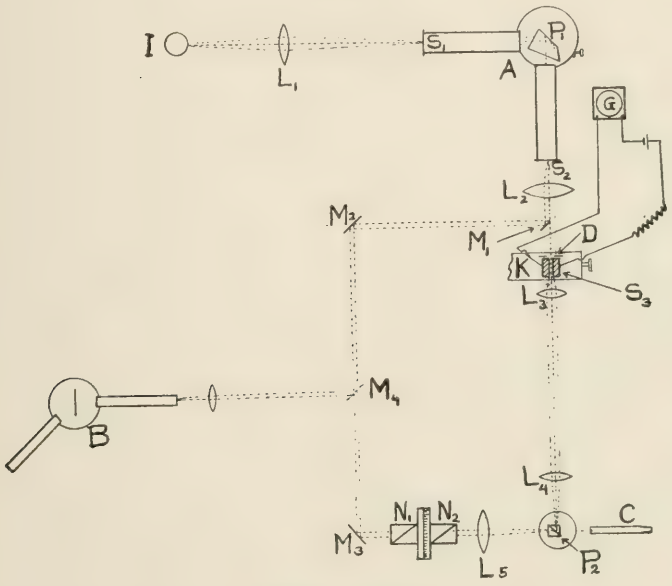


Diagram of Apparatus

the monochromatic illuminator, A. By means of prism P_1 any desired wave-length could be obtained at S_2. A lens L_2, placed so that S_2 was at its principal focus, secured parallel light at S_3. A mirror at M_1 divided the pencil into two parts, one reflected at right angles, the other passing straight through. S_3 was the slit under investigation. It was formed by two plane steel surfaces of a pair of Hoke Precision Gauges. One of the gauges was clamped securely to the bed of the interferometer, K; the other was set in wax in a frame attached to the carriage of the interferometer, brought into contact with the first gauge, and allowed to remain in that position until the wax had hardened. The slit was then placed parallel to the direction of the light. A template, D, prevented any light from passing below or above S_3. A short focus lens was placed at L_3, the center of the edge of

S_3 nearest it being at its principal focus. This pencil was then brought to a focus by L_4 on a series of parallel silver bands between two right-angled prisms, P_2. The image of the near edge of S_3, thus formed, was viewed through a low power microscope at C.

That portion of the light which was reflected by M_1 was, by means of the mirrors M_2 and M_3, made incident normally on the nicols N_1N_2, the rotation of one of which enabled one to match the intensities of the two images in P_2. By means of L_5 the light coming through N_1N_2 was focused on the silver bands in P_2. Thus there was formed at P_2 a continuous band of light; the portion *reflected from* the silver being the image of the near edge of S_3, and the portion seen *between* the silver bands being the image of S_2, formed by the light coming by way of $M_1M_2M_3$. It was found by careful trial that the results were not appreciably altered by placing the microscope, C, in line with S_2S_3; and this position being more convenient, the final measurements were taken with that arrangement. By means of an auxiliary mirror at M_4 and the spectrometer, B, the mean wave-length of the light used was determined.

Denoting by I_0 the intensity transmitted through the nicols when the angular separation of their transmission planes is zero, the intensity, I, for any other angle of separation, θ, is

$$I = I_0 \cos^2\theta \tag{1}$$

For a wide opening of S_3 the intensities of the images at P_2 were made equal by the use of a neutral tint glass in conjunction with the nicols. Letting $I_0 = 1$, since only comparative values are wanted, (1) becomes,

$$I = \cos^2\theta \tag{2}$$

After S_3 is opened to a certain width, the image at P_2 becomes constant in intensity, any further opening of S_3 simply serving to make the image wider. Hence intrinsic intensities are obtained, not total transmitted light.

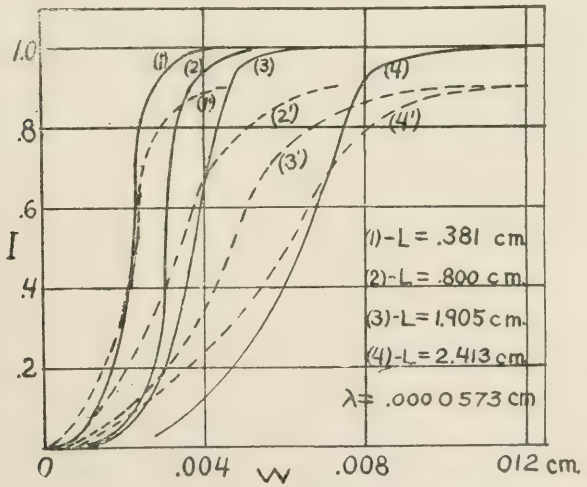
Contact of the surfaces of S_3 was determined, after thoroughly cleaning the surfaces, by the deflection of a galvanometer, G, placed in series with a dry cell, and the surfaces of S_3. Then by means of the interferometer screw the separation of the surfaces

was ascertained. For each width of S_3, three independent settings of N_2 were made, and the mean relative intensities were thus obtained.

RESULTS

The results, reduced to tabular form would occupy more space than is desirable. However, their essential character can readily be represented by a few typical curves. These are drawn to illustrate three separate relations; I as a function of w, λ and L constant (Fig. 2); I as a function of L, λ and w constant (Fig. 3); and I as a function of λ, w and L constant (Fig. 4). In the above λ represents the wave-length, w the width of slit, and L the depth of slit, all expressed in cms. The full line curves represent the experimental results: the broken line curves represent certain calculated results based on two assumptions to be discussed in the next section.

FIG. 2

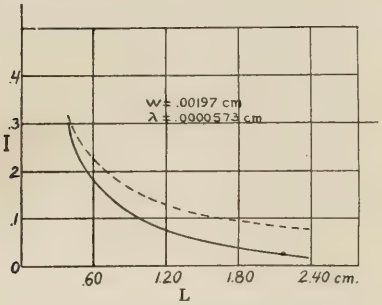


The intensity as a function of the width of slit, for various depths

THEORETICAL CONSIDERATIONS

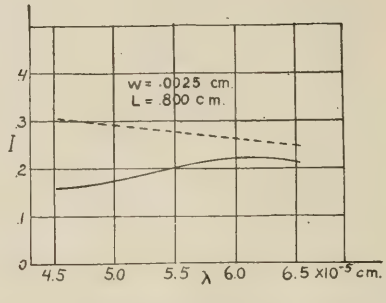
Undoubtedly the passage of a wave through such a deep channel is a complicated matter, in which diffraction, selective reflection, and possibly interference are the chief factors. An accurate test of any theory would necessitate a knowledge of the reflection

FIG. 3



The intensity as a function of the depth of slit, the width and wave length constant

FIG. 4

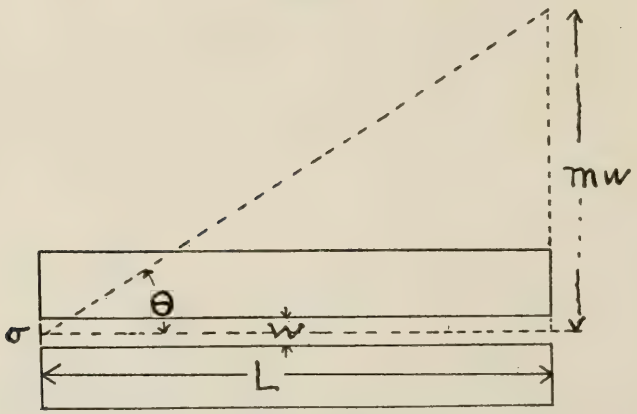


The intensity as a function of the wave-length, the width and depth constant

constants of steel (in this case) for both electric vectors, for many angles of incidence, and for all wave-lengths employed in the experimental work. Only one set of such constants, that recorded by Drude[3] for sodium light, has been found. Steel is such a variable substance, that one is in some doubt as to the validity of applying his results to the particular samples chosen for this work.

Let us assume a wave front, σ incident normally on a slit of depth L, and width w (Fig. 5). Consider a point on σ from which

FIG. 5



Section of jaws of the slit

[3] Drude. Winckelmann's Handb. der Phys., Vol. III, p. 823. 1st Ed.

a pencil is diffracted through an angle θ. The intensity of this diffracted pencil is proportional to $\sin^2 \alpha / \alpha^2$, where α is an auxiliary angle defined by the relation

$$\alpha = \frac{\pi w}{\lambda} \sin \theta \quad \ldots\ldots (3)$$

From Fig. 5 we have

$$\theta = \tan^{-1} mw/L \quad \ldots\ldots (4)$$

where we assume m to be some integral number. It can readily be shown that the number of reflections of a diffracted pencil before emergence from the slit is m. Substituting for θ in (3) its value given by (4), we obtain

$$\alpha = \frac{\pi w^2 m}{\lambda \sqrt{m^2 w^2 + L^2}} \quad \ldots\ldots (5)$$

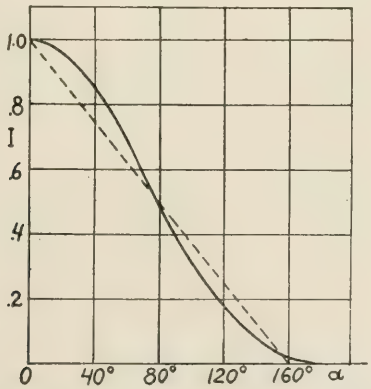
Since $m^2 w^2$ is small as compared with L^2, we may write

$$\alpha = \frac{\pi w^2 m}{\lambda L} \quad \ldots\ldots (6)$$

Negative values of θ can be neglected on account of symmetry.

Plotting the relation $I = \sin^2 \alpha / \alpha^2$, we obtain the well-known (full line) curve in Fig. 6. Beyond $\alpha = \pi$ there is a series of

FIG. 6



Diffracted intensity as a function of the auxiliary angle α

rapidly decreasing maxima, which we shall here neglect. The entire area under this curve is π, and the area under the curve from $\alpha = 0$ to $\alpha = \pi$ was calculated to be approximately $0.45\ \pi$.

If we assume the intensity of the light passing through a given slit, which is infinitely thin to be unity, the resultant intensity of light passing through a given deep slit will be given by the following expression,

$$I = \frac{1}{0.45\pi} \cdot \frac{\pi}{m} \cdot \frac{1}{2} \left[\sum_{i=o}^{i=m} \frac{\sin^2 a_i}{a_i^2} R_s^{(i)} + \sum_{i=o}^{i=m} \frac{\sin^2 a_i}{a_i^2} R_p^{(i)} \right] \quad \ldots\ldots\ldots\ldots (7)$$

where a_i represents the value of a for i reflections, R_s and R_p the reflection constants for the electric vector parallel and perpendicular respectively to the plane of incidence, and m the number of reflections when $a = \pi$. Strictly the expression (7) sums up, not a smooth curve, but a series of rectangles, which more and more closely approximate the curve as m increases. A more rigorous expression than (7) could be formulated as a definite integral, but when one realizes that the R's are not constants, but are complicated functions of the angles a, the wellnigh insuperable difficulty of evaluating the series becomes apparent. A substitution of Drude's[4] values in (7) for a typical case, yielded results that conformed in a general way with the curves of Fig. 2, but it was quite apparent that the reflection constants given by him were larger than those in the present work.

In view of the difficulty of using even (7), the following, admittedly somewhat crude method was employed. Its chief merit consists in the fact that the results obtained by it agree fairly well with the experimental results, and that the labor of computation was much abbreviated.

Then for the sake of simplicity, as stated above, let the intensity curve (Fig. 6) be replaced by a straight line running from $I = 1$ at $a = 0$, to $I = 0$ at $a = 160°$. From (6) we see that a is directly proportional to m. Hence an m can be found that will make a any desired value. Thus with a given depth of slit, L, and a given λ, we can assume various w's, and for each w we can determine a for one reflection. To obtain the number of reflections for $a = 160°$ we have merely to divide 160° by the value just determined for $m = 1$.

[4] Drude, loc. cit.

Let a represent the ordinate (Fig. 6, dotted curve) at $\alpha=0$, and m the number of reflections. The second ordinate is then, $a-a/m$; the third, $a-2a/m$, etc. The intensity is given by the areas of these strips, each multiplied by the reflecting power, R (merging R_s and R_p), raised to the power $(m-1)$. Since we wish to call the length of the base unity, the base of each strip is $1/m$. Hence the expression for the intensity is

$$I=\frac{1}{m}\left\{\frac{a+(a-a/m)}{2}+\frac{(a-a/m)+(a-2a/m)}{2}R+\cdots\cdots\right.$$
$$\left.\cdots+\frac{[a-(m-1)\,a/m]+[a-ma/m]}{2}R^{m-1}\right\}$$

which reduces to

$$I=\frac{a}{2m^2}\left\{\frac{R^{m+1}+R^m-(2m+1)R+(2m-1)}{(1-R)^2}\right\} \tag{8}$$

Applying this to the present problem we have

$$m=\frac{160\,\lambda\,L}{180\,w^2}=\frac{8\lambda L}{9\,w^2}, \qquad a=1,\ R=0.6$$

For m we must employ the nearest integer, and m should not be smaller than about 5. Substituting in (8) the values for a and R, we obtain

$$I=\frac{1}{0.32m^2}\left\{0.6^{m+1}+0.6^m+0.8\,m-1.6\right\} \tag{9}$$

For widths of slit for which m was smaller than 5, the full line intensity curve (Fig. 6) was employed, and areas were directly counted.

The broken-line curves of Figs. 2, 3, and 4 represent the results obtained as described above. In Fig. 2, (1) is the experimental curve, (1′) the corresponding theoretical curve, etc.

DISCUSSION OF RESULTS

In attempting a possible explanation of the experimental curves (Figs. 2, 3, and 4), two simple assumptions have been made; first, that the light is diffracted at the entrance to the deep slit; and second, that this diffracted light is weakened through successive reflections. These lead to results that fit the experimental curves approximately, with the exception of the case represented in Fig. 4. The differences undoubtedly arise from the rather

rough assumptions, made for the sake of simplicity. The essential character of the curves is, however, with one exception, true to experiment. Those results in which I is plotted as a function of λ cover a much smaller range of values than do those others where I is a function of w, and of L. The change of I with the wave-length is small, and the discrepancy may not be so serious as it at first appears.

In practice then, even with very thin slits, a certain width of slit must be attained before the intrinsic transmitted intensity is approximately constant. Indeed, the strong polarization of light transmitted through narrow slits is of itself sufficient evidence that at least all of one vector is completely suppressed. This result in practice probably applies only to an instrument wherein intensities are matched quantitatively by permitting light to pass through a slit of variable width, particularly when the widths are small. In such cases serious error would result.

State University of Iowa,
Jan. 1921.

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